Competition in Schedules with Traders that Neglect the Informational Content of the Price *

Anna Bayona †   Carolina Manzano ‡

April 2022

Abstract

We study a market with competition in schedules, such as in asset auctions or wholesale electricity markets, with boundedly rational sellers that partially neglect the informational content of the price. Using the cursed equilibrium concept, we find that the unique symmetric linear equilibrium with cursed sellers is more competitive, might lead to a lower trading volume, and a higher volatility of prices compared to when sellers are fully rational. Under some conditions, total surplus and profits are higher with cursed sellers than with fully rational ones. Our results critically depend on imperfect competition and on the demand elasticity.

Keywords: cursed equilibrium, bounded rationality, market power, trading volume, welfare.

JEL codes: C72, D44, D82, G14, G40

*We thank seminar participants at the Econometric Society European Winter Meeting 2021, University College Dublin and Universitat Rovira i Virgili, Marina Agranov and Xavier Vives for excellent comments. Bayona acknowledges the financial support from Banc Sabadell, the Generalitat de Catalunya (AGAUR grant 2017 SGR 640), and the Spanish Ministry of Science, Innovation and Universities (Grant: PGC2018-098670). Manzano acknowledges the financial support from the Spanish Ministry of Science and Innovation (PID2019-105982GB-I00 AEI/FEDER, UE), the Universitat Rovira i Virgili and the Generalitat de Catalunya (2019PFR-URV-B2-53). Declarations of interest: none.

†Anna Bayona, ESADE Business School, Universitat Ramon Llull, Av. Pedralbes, 60-62, 08034 Barcelona, Spain. *Tel.* +34 932 806 162, *e-mail:* anna.bayona@esade.edu

‡Carolina Manzano: Universitat Rovira i Virgili and ECO-SOS. Av. de la Universitat 1, Reus, 43204, Spain. *Tel.* (34) 977 758 914, *e-mail:* carolina.manzano@urv.cat

1 Introduction

The existence of boundedly rational traders in markets characterized by information frictions is a common phenomenon.[1] One such departure from full rationality is the (partial) inability of traders to extract the relevant information about others' private information from market prices.[2] This is especially significant in markets with incomplete information where traders compete in schedules such as in electricity markets and in dealer markets with auctions for Treasury securities and central banks' liquidity.

Our main contribution is to analyze the effects on market quality and welfare in a market where sellers neglect the informational content of the price and the demand is elastic. In contrast to perfectly competitive markets, in our setup market competitiveness is higher, the expected equilibrium price is lower, the volatility of the market price and total surplus can be higher when traders partially neglect the informational content of the price compared to markets with fully rational traders. In contrast to settings where the demand is perfectly inelastic, we find that these boundedly rational traders can cause the volatility of the market price to be higher and trading volume to be lower.

This paper presents a market with a finite number of strategic sellers that compete in schedules to supply an elastic demand based on Kyle (1989) and Vives (2011). Sellers have incomplete information about an element of their costs and receive private signals. Costs may be positively correlated among sellers because they have a common component due to events that systematically affect them all. The key behavioral element of our model is that sellers may not be fully rational in that they do not entirely extract all the information from the market price. The equilibrium concept that we use is the cursed expectations equilibrium (hereafter CEE), based on Eyster and Rabin (2005) and Eyster et al. (2019) that combines profit maximization with cursed expectations and market clearing. This modelling approach encompasses traders with different degrees of rationality: fully cursed sellers do not extract any information from the market price; partially cursed sellers infer some information; and fully rational sellers correctly and fully infer all the relevant information from the market price. For purposes of tractability, we assume that traders are symmetric in private signal precision, cost functions and cursedness.

We find that a symmetric linear CEE exists even in the case of pure common values. In a market with cursed traders, each seller has incentives to rely on her private signal because she partially neglects some information from the market price even though the price is a sufficient statistic of the joint information in the market. Hence, the Grossman-Stiglitz paradox (1980) does not work in our set-up. In addition, we find that equilibrium supply functions might be downward sloping when adverse selection is very high. This result has also been found by Vives (2011) with fully rational traders and is broadly consistent with the findings of other papers which model wholesale electricity markets, such as Holmberg and Willems (2015), Wolak (2015) and Brown and Eckert (2021).

The comparative statics results of the CEE are as follows. Suppose that costs are positively correlated and private signals are not perfectly informative. Then, with cursed sellers, supply functions are more responsive to private signals and prices compared to a market with fully

[1] See surveys by Camerer (2003), Barberis and Thaler (2005), Spiegler (2011), and Barberis (2018).

[2] See the survey by Eyster (2019) and the experimental evidence by Bayona et al. (2020).

rational sellers. Interestingly, we find that in an imperfectly competitive market, price impact decreases with the degree of cursedness. This implies a lower expected market price and a lower expected quantity supplied by sellers.

We obtain novel results with respect to the impact of cursedness on the over- or under-reaction of the market price to private signals, and hence, on the volatility of market prices. In a demand competition model, Eyster et al. (2019) find that the price under-reacts to private signals because cursed traders do not fully infer others' information from prices. As a result, the volatility of the market price reduces. Our analysis indicates that Eyster et al. (2019)'s result depends crucially on either perfect competition among traders or perfectly inelastic supply. In contrast, we study a supply competition model with strategic sellers and an elastic demand, and we find that these results might not hold because of the additional negative effect of cursedness on price impact. When the price impact effect dominates, we show that cursedness generates an over-reaction of the price on private signals, which leads to an increase in the variance of the equilibrium price. Furthermore, a higher degree of cursedness can decrease the expected trading volume in settings where supply functions are downward sloping. This overturns the main result of the related theoretical literature (Eyster et al., 2019 and Mondria et al., 2021) since in these other models trading volume is higher with behavioral traders than with fully rational traders. Our analysis shows that our results on trading volume are due to the fact that we consider an elastic demand, whereas the extant literature has considered the equivalent of a perfectly inelastic demand.

Our paper also analyzes whether cursedness is socially desirable from a welfare perspective. For this purpose, we first characterize the efficient allocation, which is the equilibrium allocation of a market with fully rational sellers that are price-takers. We then derive the expected deadweight loss at the equilibrium allocation and examine how it depends on cursedness. We find that under perfect competition, cursedness always decreases welfare. However, in a market with strategic sellers, the opposite result might arise. For example, with a perfectly inelastic demand, the expected deadweight loss is lower with cursed sellers when the correlation among costs or the degree of cursedness are sufficiently small.

Interestingly, we also find the counter-intuitive result that the expected profits of cursed sellers may actually be higher than those of fully rational sellers under some market conditions. We illustrate this in the case of a perfectly inelastic demand and show that this result occurs when the aggregate quantity is not high and cursed sellers can better align the quantity sold to the profit per unit than fully rational sellers can, i.e., sellers supply a high quantity when their profit per unit is high, and vice versa. The result that the expected profits of boundedly rational traders can be higher than those of fully rational traders has been found in other settings but for different reasons. For example, De Long et al. (1990) and Blume and Easley (1992) find that non-rational traders earn higher expected profits than rational ones by taking a disproportionate amount of risk that they themselves create. Kyle and Wang (1997) and Benos (1998) find that overconfident traders can earn higher profits than rational ones because overconfidence allows them to commit to trading more aggressively; or by inducing self-validating feedback into fundamentals (Hirshleifer et al., 2006).[3] In relation to the extant literature, our paper

[3] For a longer discussion of this issue refer to Hirshleifer (2015).

provides a novel mechanism in which boundedly rational traders can earn higher expected profits than rational traders because they can better align the quantity sold to their profit per unit.

Our paper is related to the literature which theoretically analyzes markets with competition in schedules and with rational traders (Grossman, 1981; Hart, 1985; Klemperer and Meyer, 1989; Kyle, 1989; Vives, 2011). In contrast to this literature, we introduce traders that are boundedly rational.

Both empirical (Hortaçsu and Puller, 2008) and experimental evidence (Bolle et al., 2013; Brandts et al., 2014) show important deviations from the theoretical benchmarks related to supply function competition and attribute this to players' boundedly rational behavior. Bayona et al. (2020) test the market quality implications of Vives (2011) in the laboratory and find that behavior in markets with cost correlation is very similar to behavior in markets with no cost correlation. This is because subjects fail to understand the relevant information about others' private information from market prices. In other diverse contexts, there is a growing literature which also shows that individuals fail to extract information from others' actions such as the winner's curse in common value auctions (Kagel and Levin, 2002), bilateral negotiations (Samuelson and Barzeman, 1985), acquiring a company (Charness and Levin, 2009), voting (Esponda and Vespa, 2014), and in financial markets (Ngangoué and Weizäcker, 2021). This type of behavior can be rationalized in theoretical models such as the cursed equilibrium by Eyster and Rabin (2005).

The two papers closest to ours are Vives (2011) and Eyster et al. (2019). The novelty of our paper in relation to Vives (2011) is that we allow for boundedly rational traders that only partially understand the informational content of the price. In relation to Eyster et al. (2019), we have introduced interdependent values with strategic sellers that supply an elastic demand instead of a common value setup with an inelastic supply and price-taking traders. Specifically, in a financial market context, Eyster et al. (2019) explain that the neglect of information derived from prices can explain excessive trade volume. Specifically, they show that cursedness produces more trade than would occur if all traders were fully rational. They also find that cursedness makes prices under-react to private signals, which implies positively autocorrelated price changes. In contrast, we find that when the demand is elastic, an increase in the degree of cursedness might decrease trading volume. Moreover, we also show price underreaction to private signals due to cursedness under either perfect competition or in markets with a perfectly inelastic demand. To find price overreaction, we have to restrict our attention to a setup with strategic behavior and an elastic demand. In a related paper, Mondria et al. (2021) study a two period model where traders first select the optimal amount of sophistication, and then trade in a financial market.[4] They argue that since interpreting financial prices is costly this generates endogenous noise trading. Costly interpretation leads to price momentum, excess return volatility, and excess trading volume. In contrast, we find that trading volume does not necessarily increase with cursedness and, in addition, we evaluate the welfare effects.

This paper is organized as follows. Section 2 presents the model. Section 3 characterizes

[4] In contrast, we consider that the degree of cursedness is not rationally chosen since this is a behavioral bias inherent to market participants due to their limited capacity to process information.

the market equilibrium with cursed traders and derives some comparative statics. Section 4 analyzes the implications for market quality. Section 5 examines the effects of cursedness on total surplus and expected profits. Section 6 presents the concluding remarks. Proofs can be found in the paper's Appendix.

2 Model

We consider a market with N symmetric suppliers that sell a homogeneous good, $N > 1$. Seller i has the following cost for supplying x_i units of the good

$$C(x_i; \theta_i) = \theta_i x_i + \frac{\lambda}{2} x_i^2,$$

where θ_i is a random parameter, and λ represents a transaction cost parameter, with $\lambda \geq 0$. We assume that $\theta_i \sim N(\overline{\theta}, \sigma_\theta^2)$, and that $cov(\theta_i, \theta_j) = \rho\sigma_\theta^2$, for $j \neq i$, such that the correlation coefficient $\rho \in [0, 1]$. This framework encompasses common values if $\rho = 1$, private values if $\rho = 0$, and interdependent values if $0 < \rho < 1$.

Prior to trading, each seller receives a private signal, s_i, about her random cost parameter θ_i, such that $s_i = \theta_i + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. We assume that error terms in the private signals are correlated neither with themselves nor with the cost random parameters, i.e., $cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$, and $cov(\varepsilon_i, \theta_j) = 0$ for all i and j. Define the average signal of all sellers as $\widetilde{s} \equiv \frac{\sum_{j=1}^N s_j}{N}$.

In terms of the demand side, suppose that there is a continuum $[0, 1]$ of consumers. There is a representative consumer, indexed by k, with the following quasi-linear utility function $U(q_k, O_k) = \alpha q_k - \beta q_k^2/2 + O_k$, where α and β are positive parameters and q_k denotes the consumer's consumption of the good and O_k is a composite measure of the consumer's consumption of all other goods. Let p denote the price of the good, m the consumer's income, and the composite good's price be normalized to 1. Maximizing $U(q_k, O_k)$ subject to the budget constraint that $pq_k + O_k \leq m$ gives the individual inverse demand function: $p = \alpha - \beta q_k$, and hence, the (aggregate) inverse demand function satisfies: $p = \alpha - \beta Q$, where $Q = \int_0^1 q_k dk$. Throughout the paper we assume that $\alpha > \overline{\theta}$ and consider the two special types of demand: perfectly inelastic demand, which can be incorporated as $\beta \to \infty$ and $\frac{\alpha}{\beta} \to Q$; and perfectly elastic demand, which is represented by the limit when $\beta \to 0$.

Denote by X_i the strategy for seller i, which is a mapping from the signal space to the space of supply functions. Thus, $X_i(s_i, \cdot)$ represents a supply function for seller i who observes s_i.

The timing of the game is as follows. At $t = 0$, the random parameters $\{\theta_i\}_{i=1,\ldots,N}$ are drawn but not observed. At $t = 1$, each player observes her own private signal and submits a supply schedule. At $t = 2$ the market clears: supply functions are aggregated and crossed with the demand to obtain an equilibrium price. Finally, profits are collected.

Following Eyster and Rabin (2005) and Eyster et al. (2019), we use the Cursed Expectations Equilibrium (CEE) concept, which combines profit maximization with cursed expectations and market clearing.

Definition: *A set of supply functions* $\{X_i(s_i,\cdot)\}_{i=1,...,N}$ *is a CEE if and only if the following two statements hold:*
(*i*) (*Optimization*): *For each seller* i, $i=1,...,N$, *each realization of the private signal* (s_i) *and the price* p, $X_i(s_i,p)$ *maximizes her expected pay-off*

$$(1-\chi)\mathbb{E}[\pi_i \mid s_i, p] + \chi\mathbb{E}[\pi_i \mid s_i],$$

where χ *is sellers' degree of cursedness, and* π_i *are profits for seller* i *given by*

$$\pi_i = pX_i(s_i,p) - C\left(X_i(s_i,p);\theta_i\right),$$

and each seller takes as given the strategies of other suppliers and considers the effect of the quantity she supplies on the price.
(*ii*) (*Market Clearing*): *For each* s_i, $i=1,...,N$, *the market price,* p, *satisfies*

$$\sum_{j=1}^{N} X_j(s_j,p) = \frac{\alpha - p}{\beta}.$$

Notice that the optimization process is the arithmetic average of the expected profits of sellers who condition on both the private signal and the price (fully rational), and the expected profits of sellers who only condition on the private signal and completely ignore the informational content of the price (fully cursed).[5] Depending on sellers' degree of cursedness, χ, which is common knowledge, we can distinguish the following cases: if $\chi=0$, then sellers are fully rational; if $\chi=1$, then sellers are fully cursed; and if $0<\chi<1$, then sellers are partially cursed.[6]

Our attention will be focused on *symmetric linear CEE*, in which strategies are linear and identical among sellers (for short, equilibria). Thus, in equilibrium the supply functions can be written as

$$X(s_i,p) = b - as_i + cp,\ i=1,...,N,$$

where b, a, and c are constant.

3 Equilibrium characterization

Consider seller i. Given that other sellers, $j \neq i$, use the strategy $X(s_j,p) = b - as_j + cp$, seller i faces the following residual inverse demand

$$p = \alpha - \beta\left((N-1)b - a\sum_{j\neq i} s_j + (N-1)cp + x_i\right).$$

[5] For tractability purposes, Eyster et al. (2019) use the geometric average of the expected utility of traders who condition on both the private signal and the price, and of traders who only condition on the private signal. We use the arithmetic average as in Eyster and Rabin (2005).
[6] When $\chi=0$ our model is equivalent to Vives (2011).

Provided that $1+\beta(N-1)c \neq 0$, this residual inverse demand can be rewritten as $p = I_i - dx_i$, with $I_i = \frac{\alpha-\beta\left((N-1)b-a\sum_{j\neq i}s_j\right)}{1+\beta(N-1)c}$, and the slope of the inverse residual demand (in absolute terms), which is price impact and is equal to

$$d = \frac{\beta}{1+\beta(N-1)c}. \tag{1}$$

Note that the pair (s_i, p) is informationally equivalent to the pair (s_i, I_i). Hence, seller i chooses x_i to maximize

$$(1-\chi)\left(\left(I_i - dx_i\right)x_i - \mathbb{E}\left[\theta_i|s_i, I_i\right]x_i - \frac{\lambda}{2}x_i^2\right) + \chi\left(\left(I_i - dx_i\right)x_i - \mathbb{E}\left[\theta_i|s_i\right]x_i - \frac{\lambda}{2}x_i^2\right).$$

The first order condition of this optimization problem is

$$(1-\chi)\left(I_i - \mathbb{E}\left[\theta_i|s_i, I_i\right] - 2dx_i - \lambda x_i\right) + \chi\left(I_i - \mathbb{E}\left[\theta_i|s_i\right] - 2dx_i - \lambda x_i\right) = 0. \tag{2}$$

The second order sufficient condition for a maximum implies $2d+\lambda > 0$. Thus, the optimal supply function for a seller i is given by

$$X(s_i, p) = \frac{p - (1-\chi)\mathbb{E}\left[\theta_i|s_i, p\right] - \chi\mathbb{E}\left[\theta_i|s_i\right]}{d+\lambda}. \tag{3}$$

The next proposition characterizes the equilibrium.

Proposition 1: *Let $\rho \neq 1 + \frac{\chi\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2(1-\chi)}$ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \infty$, and $\lambda > 0$. Then, there exists a unique equilibrium, where*

$$a = \frac{\chi\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2} + (1-\chi)\frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}, \tag{4}$$

$$b = \frac{\frac{\alpha M}{\beta N} - \frac{\chi\frac{\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2}+(1-\chi)\frac{\sigma_\varepsilon^2}{(1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}\overline{\theta}}{1+M}, \tag{5}$$

$$c = \frac{\frac{1}{d+\lambda} - \frac{M}{\beta N}}{1+M}, \tag{6}$$

and

$$M = \frac{(1-\chi)\,\rho N\sigma_\varepsilon^2}{\left(1-\rho+\chi\frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)} \tag{7}$$

is an index of adverse selection. Price impact, d, is given by

$$d = \frac{\beta N\left(M-N+2\right) - \lambda\left(M+N\right) + \sqrt{\beta^2N^2\left(M-(N-2)\right)^2 + \lambda^2\left(N+M\right)^2 + 2\beta\lambda N\left(N+M\right)^2}}{2\left(N+M\right)},$$

and the equilibrium price is

$$p = \frac{\alpha - \beta N b}{1+\beta N c} + A\widetilde{s}, \tag{8}$$

where

$$A = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2} + (1-\chi)\frac{\rho\sigma_\theta^2\sigma_\varepsilon^2(N-1)}{(\sigma_\theta^2+\sigma_\varepsilon^2)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}}{\frac{d+\lambda}{N\beta}+1}. \tag{9}$$

Remark 1: If $\lambda = 0$, then an equilibrium exists provided that $M - N + 2 > 0$. Price impact is then $d = \frac{\beta N(M-N+2)}{M+N}$, and the rest of the supply function coefficients are given by substituting this value of d into equations (4), (5), and (6).

Remark 2: If $(1-\chi)\rho\sigma_\varepsilon^2 = 0$ (i.e., suppliers are fully cursed, private signals are perfectly informative or random parameters are uncorrelated), then suppliers do not learn about their cost parameters from prices. Hence, $\mathbb{E}[\theta_i|s_i,p] = \mathbb{E}[\theta_i|s_i]$ for $i = 1,...,N$. The supply functions are given by $X(s_i,p) = \frac{p-\mathbb{E}[\theta_i|s_i]}{d+\lambda}$, and the equilibrium coefficients satisfy $a = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}$, $b = -\frac{\frac{\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}\overline{\theta}$ and $c = \frac{1}{d+\lambda}$. Using (1), it follows that $d = \frac{-\beta(N-2)-\lambda+\sqrt{\beta^2(N-2)^2+2\beta\lambda N+\lambda^2}}{2}$. Note that in this case $d+\lambda > 0$ requires $\lambda > 0$.

Remark 3: Suppose that $(1-\chi)\rho\sigma_\varepsilon^2 > 0$. Note that the expression of M requires that $1 - \rho + \chi\frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2} \neq 0$. If sellers are fully rational ($\chi = 0$), then the previous condition becomes $\rho \neq 1$, and hence, an equilibrium exists provided that $\rho < 1$. Otherwise, i.e., if $\chi > 0$, then $1 - \rho + \chi\frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2} > 0$, and therefore, the equilibrium always exists even in the case of perfect positive correlation. Note that in a framework with common values ($\rho = 1$), where sellers receive private signals that are not perfectly informative ($\sigma_\varepsilon^2 > 0$) and are partially or fully cursed ($\chi > 0$), they give some weight on their private signals, even though the equilibrium price reveals all the information available in the market. Thus, we conclude that in our model the Grossman and Stiglitz (1980) paradox does not work.

From Proposition 1, it emerges that the equilibrium supply function of seller i is *not* a convex combination of the fully rational equilibrium supply function ($\chi = 0$) and the fully cursed equilibrium supply function ($\chi = 1$) because price impact depends on cursedness through the index of adverse selection. This leads to interesting results for market quality and welfare.

As shown in equation (8), the coefficient A can be interpreted as the weight of the average signal on the equilibrium price. This allows us to determine whether the price under-reacts or over-reacts to the average signal in relation to the fully rational case.

The next corollary provides comparative statics results concerning how several underlying parameters affect the equilibrium coefficients and price impact.

Corollary 1: *Suppose that* $\rho\sigma_\varepsilon^2(1-\chi) > 0$ *and* $\lambda > 0$.
(*i*) *An increase in the correlation among random cost parameters,* ρ, *makes sellers' supply functions less responsive to private signals and prices* (*lower* a *and* c, *respectively*) *and increases price impact,* d.
(*ii*) *An increase in the degree of cursedness,* χ, *makes sellers' supply functions more responsive to private signals and prices* (*higher* a *and* c, *respectively*) *and decreases price impact,* d.
(*iii*) *An increase in the slope of the inverse aggregate demand function,* β, *increases price impact, which makes sellers' response to the private signal decrease* (*lower* a). *Sellers' response to the*

price, c, increases with β except if $M < N < \widetilde{N}$ and β is high enough, where c decreases with β.

Remark 4: Note that, from Remark 2, we have that (i) when $\sigma_\varepsilon^2 (1 - \chi) = 0$ the equilibrium coefficients are independent of ρ; (ii) when $\rho\sigma_\varepsilon^2 = 0$ (i.e., the random costs parameters are uncorrelated or the private signals are perfectly informative), the equilibrium coefficients are independent of cursedness because in such a case prices are not useful for sellers when predicting their random costs parameters. Moreover, in this case the equilibrium coincides with the full information equilibrium; (iii) when $\rho\sigma_\varepsilon^2 (1 - \chi) = 0$, an increase in β raises price impact, which makes sellers' supply functions less responsive to private signals and prices (lower a and c, respectively).

The first part of Corollary 1 analyzes the relationship between cost interdependence and the equilibrium supply function. From equation (3), we deduce that a high price has a direct effect to increase the quantity supplied by a seller, but also provides information about sellers' costs. When $\rho\sigma_\varepsilon^2 (1 - \chi) > 0$, a high price conveys the information to a seller that the costs of her rivals are high, and therefore, that her own costs are high because of the positive correlation between cost parameters (adverse selection effect). An increase in the correlation among costs raises the adverse selection effect. Therefore, an increase in the price makes a seller more pessimistic about her cost parameter, which leads to a lower increase in the quantity supplied due to the change in the price. Notice that when the adverse selection effect is very high, i.e., $M > \frac{N\beta}{\beta+\lambda}$, then supply functions are downward sloping as they can be in Vives (2011). Thus, c decreases due to an increase in cost correlation, which implies that price impact increases. The increase in price impact jointly with the fact that seller i relies less on her private signal when predicting θ_i, leads seller i to reduce her supply sensitivity to the private signal when ρ increases.

Importantly, we also find that increasing cursedness makes supply functions flatter in relation to when sellers are fully rational. This is because cursed sellers, by partially ignoring the informational content of the price, reduce the adverse selection effect, which results in an increase in the equilibrium coefficient c, and hence, a decrease in price impact. When cursedness increases, the reduction in price impact jointly with the fact that seller i relies more on her private signal in predicting θ_i leads to a supply function with higher response to the private signal.

With respect to how the slope of the inverse aggregate demand function, β, affects the equilibrium supply function, we find the following. A higher β increases price impact, and consequently, sellers' response to the private signal decrease. The effect of increasing β on c is ambiguous since, as shown in (6), it has a direct effect of increasing the supply function slope but it also has an indirect effect of increasing price impact, which reduces the supply function slope. We characterize the conditions under which one effect dominates the other and find that it depends on the index of adverse selection, the transaction cost parameter, the number of sellers, and the slope of the inverse aggregate demand function.

3.1 Equilibrium characterization in the benchmark models

In this subsection, we provide two benchmark models which allow us to disentangle the effects that we find in the subsequent sections.

3.1.1 Perfect competition

In this part we focus on a perfectly competitive market, where each seller takes the price as given when making her output decision. As each seller ignores her effect on prices, price impact is zero, i.e., $d = 0$. Hence, from Proposition 1, we can characterize this equilibrium as follows:

Corollary 2: *Let* $\rho \neq 1 + \frac{\chi\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2(1-\chi)}$, $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \infty$, *and* $\lambda > 0$. *There exists a unique equilibrium where price impact is* $d = 0$, *and the supply function coefficients are given by substituting this value of* d *into equations* (4), (5), *and* (6).

3.1.2 Perfectly inelastic demand

The demand is perfectly inelastic demand when $\beta \to \infty$ and $\frac{\alpha}{\beta} \to Q$. For this benchmark model, we obtain the following equilibrium result.

Corollary 3: *Let* $\rho \neq 1 + \frac{\chi\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2(1-\chi)}$, $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \infty$, *and* $\lambda > 0$. *If the demand is perfectly inelastic, then the equilibrium exists provided that* $M - N + 2 < 0$. *In such a case* $d = \frac{\lambda(1+M)}{N-2-M}$, *and the supply function coefficients are given by substituting this value of* d *into equations* (4), (5), *and* (6), *and taking the limits when* $\beta \to \infty$ *and* $\frac{\alpha}{\beta} \to Q$.

Notice that if the demand is perfectly inelastic then supply functions are always upward sloping and suppliers can influence the residual demand ($d > 0$). However, in the case of a perfectly elastic demand ($\beta \to 0$) in equilibrium sellers behave as price takers ($d \to 0$) and we obtain the same result as in Corollary 2.

4 Market quality

Using the equilibrium derived in Section 3, we analyze the implications of cursedness on the following dimensions of market quality: competitiveness, expected equilibrium price and its volatility, trading volume, and price informativeness. We first present the results for the general model, and then, derive the results in the benchmark models of sub-section 3.1 in order to understand what explains our findings. For all the results of this section, we assume that $(1-\chi)\,\rho\sigma_\varepsilon^2 > 0$ and $\rho \neq 1 + \frac{\chi\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2(1-\chi)}$.

4.1 Competitiveness

We first study how cursedness affects the market's competitiveness. We have already seen in Corollary 1(ii) that price impact decreases in cursedness under imperfect competition. Here, we explore how another measure of competitiveness – the expected price-cost margin (the difference between the market price and a seller's marginal cost, in expected terms)– is affected by cursedness. From the expression of the cost function and equation (3), we have that the expected price-cost margin is equal to $\mathbb{E}[dx_i]$, with

$$\mathbb{E}[x_i] = \frac{\alpha - \overline{\theta}}{d + \lambda + N\beta}, \tag{10}$$

where x_i denote the equilibrium quantity supplied by seller i. This leads to the following proposition:

Proposition 2: *With imperfect competition, a higher degree of cursedness increases competitiveness.*

The proposition follows because increasing cursedness decreases price impact, which decreases the expected price-cost margin. Proposition 2 also holds in the limiting case when the demand is perfectly inelastic. However, with perfect competition cursedness does not affect the competitiveness of markets since price impact is always zero.

4.2 Expected equilibrium price and its volatility

The following two results are dedicated to the relationship between cursedness and the expected equilibrium price and its volatility. First, taking expectation in (8) and using Proposition 1, it follows that the expected equilibrium price, $\mathbb{E}[p]$, is

$$\mathbb{E}[p] = \alpha - N\beta \left(\frac{\alpha - \overline{\theta}}{d + \lambda + N\beta} \right). \tag{11}$$

We notice that the effect of cursedness on the expected equilibrium price is through price impact.

Second, with respect to the volatility of the market price, $var[p]$, we note from (8) that

$$sign \left(\frac{\partial var[p]}{\partial \chi} \right) = sign \left(\frac{\partial A}{\partial \chi} \right). \tag{12}$$

Hence, understanding how cursedness changes the weight of the average signal on the price, A, is useful to characterize how the variance of the market price changes with cursedness. In addition, from equation (9), we find that a change in the degree of cursedness has two effects on A: a direct effect because A depends on χ explicitly and, and an indirect effect through price impact. We find that the direct effect is negative since A decreases in χ, implying that, with cursed traders, the market price under-reacts to private signals in relation to when traders are fully rational. Specifically, when sellers are fully cursed ($\chi = 1$), they do not condition on the market price, meaning that seller i's private signal receives no weight in other sellers' conditional expectations. This leads to a lower A with fully cursed sellers compared to fully rational sellers. The same reasoning applies with partial cursed sellers. With respect to the indirect effect, we find that it is positive since increasing cursedness reduces price impact, which results in an increase in A. Hence, both effects go in opposite directions. Consequently, in the general model, the total effect of cursedness on the weight of the average signal on the equilibrium price and on the volatility of prices depends on market conditions as shown in the next proposition.

Proposition 3: *In the general model:*

(i) *Increasing the degree of cursedness decreases the expected equilibrium price.*
(ii) *Increasing the degree of cursedness decreases the variance of the market price if* $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \geq \frac{N^2\beta}{\lambda}$. *Otherwise, var* $[p]$ *can increase with cursedness if:*

either $\rho > \widehat{\rho}$ *and* $\chi \leq \widehat{\chi}$, *where* $\widehat{\rho}$ *is the solution of* $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda}$ *and* $0 < \widehat{\chi} < 1$,

or $\rho \geq \widehat{\widehat{\rho}} > \widehat{\rho}$ *and* $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \leq \frac{1}{\beta\lambda}\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2$, *where* $\widehat{\widehat{\rho}}$ *is the solution of* $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=1}\right)^2}{\beta\lambda}$.

We then apply these results to the benchmark models.

Corollary 4: (i) *If the demand is perfectly inelastic, then increasing the degree of cursedness decreases the expected equilibrium price. By contrast, under perfect competition, cursedness does not affect the expected equilibrium price.*

(ii) *With either perfect competition or with a perfectly inelastic demand, increasing the degree of cursedness decreases the volatility of the market price.*

We find that, in imperfectly competitive markets, the expected equilibrium price decreases with the degree of cursedness. This is because an increase in cursedness decreases price impact, which decreases the expected equilibrium price. By contrast, with perfect competition price impact is zero, and therefore, the expected price is independent of the degree of cursedness.

With respect to the variance of prices, we first focus on the benchmark models (Corollary 4). With perfect competition, the indirect effect of cursedness on A is non-existent since price impact is zero. Therefore, the variance of prices always decreases with cursedness. Similarly, with a perfectly inelastic demand, the indirect effect becomes negligible, and we obtain the same result that the variance of prices decreases with cursedness. This result is consistent with Eyster et al. (2019) since their paper considers a market with a finite number of cursed price-takers that face an inelastic supply function.

Proposition 3 characterizes the sign of a change in the degree of cursedness on the variance of prices or, equivalently, on the weight of the average signal on the price (A) because of (12). In the general model, we find that A decreases with cursedness (as in the benchmark models) when the indirect effect has little significance. For instance, if β is low enough, then price impact is also very low independently of the degree of cursedness. Consequently, a change in the degree of cursedness has a small effect on d, which results in an indirect effect of little relevance. In markets where β is not low, for intermediate values of $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ small or χ high, then the change in price impact due to a change in cursedness is relatively small and, consequently, the indirect effect is dominated by the direct effect. The opposite result occurs, that is that A increases with cursedness, when the indirect effect is sufficiently large so that it dominates the direct effect. We show that this occurs for intermediate or low values of $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$, ρ large enough and χ low; or when $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ is small enough and ρ is large enough.[7]

[7] To understand the last parameter configuration note that M decreases with $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ when ρ is large enough. Therefore, adverse selection is relevant when $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ is low provided that ρ is large enough.

The fact that the variance of prices can increase with cursedness overturns the main result of the extant literature (Eyster et al., 2019) since our result implies that the price can over-react to private signals with cursed traders in relation to when all of them are fully rational. This is due to the indirect effect which is relevant in our set-up since it considers imperfect competition and an elastic demand, while Eyster et al. (2019) consider a non-strategic traders and the equivalent of an inelastic demand (as in our benchmark models).

4.3 Trading volume

The trading volume that seller i generates is the expected absolute value of the quantity supplied by seller i, i.e., $\mathbb{E}(|x_i|)$, which is given by

$$\mathbb{E}\left[|x_i|\right] = \sqrt{\frac{2}{\pi}} var\left[x_i\right] e^{-\frac{(\mathbb{E}[x_i])^2}{2(var[x_i])^2}} + \frac{2\mathbb{E}\left[x_i\right]}{\sqrt{\pi}} \left(\int_{\frac{-\mathbb{E}[x_i]}{\sqrt{2}var[x_i]}}^{0} e^{-y^2} dy \right),$$

and depends positively on both the expected value and the variance of the quantity supplied by a seller, denoted by $\mathbb{E}\left[x_i\right]$ and $var[x_i]$, respectively. The next lemma analyzes how $\mathbb{E}\left[x_i\right]$ and $var[x_i]$, vary with cursedness.

Lemma 1: *In the general model, increasing the degree of cursedness:*
(i) increases the expected quantity supplied by each seller.
(ii) increases the variance of the quantity supplied by each seller if adverse selection is not too large. Specifically, if supply functions are upward sloping, then $var[x_i]$ increases with the degree of cursedness. Otherwise, if adverse selection is large, the variance of the quantity supplied may decrease with cursedness.

Applying this result to the benchmark models we obtain

Corollary 5: *If the demand is perfectly inelastic, then increasing the degree of cursedness increases the expected value and variance of the quantity supplied.*

In Lemma 1(i) we find that in imperfectly competitive markets, increasing the degree of cursedness decreases price impact, which increases the expected quantity supplied by seller i by equation (10). However, in perfectly competitive markets, the expected quantity supplied is not affected by the degree of cursedness since price impact is null.

Concerning the result of Lemma 1(ii), note that the equilibrium quantity supplied by seller i is given by

$$x_i = \frac{b + c\alpha}{1 + \beta N c} + a\left(\widetilde{s} - s_i\right) - \frac{A}{N\beta}\widetilde{s}, \tag{13}$$

and its variance is

$$var[x_i] = a^2 \left(\frac{N-1}{N} \left(\left(1-\rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2 \right) \right) + \frac{A^2}{(N\beta)^2} \left(\frac{\sigma_\theta^2(1+(N-1)\rho) + \sigma_\varepsilon^2}{N} \right). \tag{14}$$

Thus, an increase on the degree of cursedness has two effects on the variance of the quantity supplied: (1) a positive effect since sellers increase their response to the private signal a, which increases the variance of the quantity supplied; (2) an ambiguous effect of cursedness on the weight of the average signal on the price, A, characterized by Proposition $A.1$ included in the Appendix. If A increases with the degree of cursedness, then the overall effect is positive, and hence, the variance of the quantity supplied increases with cursedness. However, if A decreases with cursedness (if β is low enough or if β is not low, for intermediate values of $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ small or χ high), and if this effect dominates then the variance of the quantity supplied decreases with cursedness.

With a perfectly inelastic demand, we find that the variance of the quantity supplied increases with cursedness. This is because if $\beta \to \infty$, then $\frac{A}{N\beta}$ vanishes, and this implies that the second term in (14) also converges to zero. Thus, the variance of the quantity supplied depends on cursedness only through the sellers' response to the private signal, which implies that increasing the degree of cursedness increases the variance of the quantity supplied.

Next, using the results of Lemma 1 we can characterize how trading volume changes with the degree of cursedness.

Proposition 4: *In the general model, increasing the degree of cursedness increases trading volume if $\alpha - \overline{\theta}$ is high enough or if adverse selection is not too large. Otherwise, trading volume may decrease with cursedness.*

We now apply this proposition to the benchmark model of perfectly inelastic demand.

Corollary 6: *If the demand is perfectly inelastic, then increasing cursedness increases trading volume.*

Recall that trading volume depends positively on both the expected value and variance of the quantity supplied. Proposition 4 shows that increasing the degree of cursedness increases trading volume if the expected value dominates or if the variance of the quantity supplied also increases with cursedness. The expected value dominates when the difference between the intercept of the inverse demand and the expected value of the cost random parameters is large enough. However, if $\alpha - \overline{\theta}$ is sufficiently low and β is very small or β is not low, for intermediate values of $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ small or χ high so that supply functions are downward sloping because adverse selection is very high, then we may obtain that trading volume decreases with cursedness. Note that this can also happen in perfectly competitive markets. This is because increasing cursedness makes sellers react less to the market price (since c tends to zero), which leads to a reduction in trading volume.

To sum up, if the slope of the demand function, β, is small or if β is not low, for intermediate values of $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ small or χ high, then we find that trading volume may decrease with cursedness, and this contrasts with the results derived by Eyster et al. (2019) and Mondria et al. (2021), which consider similar models with a perfectly inelastic demand.[8] However, when the demand is perfectly inelastic, our result coincides with theirs: trading volume always increases with cursedness.

[8] In Eyster et al. (2019) and Mondria et al. (2021) traders submit demand functions and there is an inelastic supply. This can be shown to be equivalent to our setting with agents submitting supply functions with an inelastic demand.

4.4 Price informativeness

We consider two measures of price informativeness. The first is the classic one used in the literature adapted for interdependent values: $\tau = (var[\theta_i|p])^{-1}$. The second is based on a measure used by Rostek and Weretka (2012), and is given by $\psi = 1 - \frac{var[\theta_i|s_i,p]}{var[\theta_i|s_i]}$. Using both measures, we obtain the following proposition.

Proposition 5: *Price informativeness is independent of cursedness.*

The result follows because we can see from the equilibrium price equation (8) that the market price is a linear function of the average signal. This implies that the price contains the same information as the average of all sellers' private signals, and hence, price informativeness is independent of cursedness.

5 Welfare Analysis

In this section, we address the normative question of whether cursedness is socially desirable. We compare the equilibrium allocation to an efficiency benchmark, and we also analyze whether expected profits can be higher with cursed traders than with fully rational traders.

5.1 Total surplus

In accordance with the related literature (Vives, 1988; Angeletos and Pavan, 2007), we define an efficient allocation as the solution to a planner's problem that consists of maximizing expected total surplus under the constraint that sellers use decentralized linear production strategies in their private signals and the price (or equivalently $\widetilde{s}$). Formally, an efficient allocation solves the following optimization program:

$$\underset{\substack{\widehat{a},\widehat{b},\widehat{c} \\ s.t. \quad x_i=\widehat{b}-\widehat{a}s_i+\widehat{c}\widetilde{s},\ i=1,...,N}}{max} \mathbb{E}\left[TS\left(x\right)\right] \qquad ,$$

where $TS\left(x\right)$ denotes the total surplus corresponding to an allocation $x = (x_1, ..., x_N)'$ and is given by

$$TS\left(x\right) = \int_0^X p(Z)dZ - \sum\nolimits_{i=1}^{N} C\left(x_i;\theta_i\right),$$

with $X = \sum_{i=1}^{N} x_i$.

In the next proposition we solve for the efficient allocation and derive the expression of the expected equilibrium deadweight loss, denoted by $\mathbb{E}\left[DWL\right]$, which is the difference between the expected total surplus at the efficient and equilibrium allocations. To this end, for a given vector of private signals $s = (s_1, ..., s_N)'$, we rewrite the equilibrium allocation of equation (13) parameterized by price impact (d) and sellers' degree of cursedness (χ) as

$$x_i\left(s;d,\chi\right) = \frac{\alpha - \overline{\theta}}{d+\lambda+N\beta} + a(d,\chi)\left(\widetilde{s}-s_i\right) - \frac{A(d,\chi)}{N\beta}\left(\widetilde{s}-\overline{\theta}\right), \tag{15}$$

$i=1,...,N$, where $a(d,\chi)$ and $A(d,\chi)$ represent the equilibrium coefficient and the weight of the average signal on the price as functions of price impact and sellers' degree of cursedness.

Proposition 6.
(i) *The efficient allocation is the equilibrium allocation of a market where sellers are fully rational and behave as price-takers, i.e.,* $x(s;0,0)$.
(ii) *The expected deadweight loss at the equilibrium allocation is given by*

$$\mathbb{E}[DWL] = \frac{N(N\beta+\lambda)}{2}\mathbb{E}\left[(\widetilde{x}(s;0,0)-\widetilde{x}(s;d,\chi))^2\right] + \frac{\lambda}{2}\sum_{j=1}^{N}\mathbb{E}\left[(u_j(s;0,0)-u_j(s;d,\chi))^2\right], \tag{16}$$

with $\widetilde{x}(s;d,\chi) = \frac{\sum_{i=1}^{N} x_i(s;d,\chi)}{N}$, $\widetilde{x}(s;0,0) = \frac{\sum_{i=1}^{N} x_i(s;0,0)}{N}$, $u_j(s;d,\chi) = x_j(s;d,\chi) - \widetilde{x}(s;d,\chi)$, *and* $u_j(s;0,0) = x_j(s;0,0) - \widetilde{x}(s;0,0)$, $j=1,...,N$.

In Proposition 6(i), we prove that the efficient allocation corresponds to the allocation of the price-taking equilibrium ($d=0$) in a market where all the sellers are fully rational ($\chi=0$). In Proposition 6(ii), we show that the expected deadweight loss at the equilibrium allocation can be decomposed in two components. The first represents aggregate inefficiency which occurs because the aggregate quantity produced in the market ($N\widetilde{x}(s;d,\chi)$) is distorted in relation to the efficient outcome ($N\widetilde{x}(s;0,0)$), while sellers produce in a cost-minimizing way. The second component is distributive inefficiency, which is on account of a distortion in the distribution of production for a given average quantity $\widetilde{x}$.[9] The next proposition shows the results of how expected deadweight loss and its components depend on cursedness.

Proposition 7. *Suppose that* $\rho\sigma_\varepsilon^2(1-\chi)>0$ *and* $\lambda>0$.
(i) *Aggregate inefficiency decreases with cursedness when* $\alpha-\overline{\theta}$ *is high enough. Otherwise, aggregate inefficiency may increase with cursedness.*
(ii) *Distributive inefficiency decreases with cursedness if* $\rho \le \widehat{\rho}_{DI} < 1$ *or if* $\widehat{\rho}_{DI} < \rho < 1$ *and* χ *is low enough.*[10] *Otherwise, distributive inefficiency increases with cursedness.*
(iii) *Expected deadweight loss decreases with cursedness if: a)* $\alpha-\overline{\theta}$ *is high enough and* $\rho \le \widehat{\rho}_{DI} < 1$*; b)* $\alpha-\overline{\theta}$ *is high enough,* $\widehat{\rho}_{DI} < \rho < 1$ *and* χ *is low enough. Otherwise, expected deadweight loss may increase with cursedness.*

In the corollary below we specify these results to each of the benchmark models.

Corollary 7. *Suppose that* $\rho\sigma_\varepsilon^2(1-\chi)>0$ *and* $\lambda>0$.
(i) *With perfect competition, expected deadweight loss increases with cursedness.*
(ii) *If the demand is perfectly inelastic, the expected deadweight loss decreases with cursedness if* $\rho \le \frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$ *or if* $\rho > \frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$ *and* χ *is low enough. Otherwise, expected deadweight loss increases with cursedness.*

In perfectly competitive markets, both aggregate and distributive inefficiencies increase with cursedness, and therefore, the expected deadweight loss increases with cursedness as shown in

[9] Refer to Vives (2011) for further details on the derivation of this decomposition.

[10] The value $\widehat{\rho}_{DI}$ belongs to $[0,1]$ and is the solution of $d=\frac{\lambda\chi\rho\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)(1-\rho)}$. In such a case $a(0,0)=a(d,\chi)$.

Corollary 7(i). The rationale is as follows. With respect to aggregate inefficiency, from equation (15), we have

$$\widetilde{x}\left(s;0,0\right)=\frac{\alpha-\overline{\theta}}{\lambda+N\beta}-\frac{A(0,0)}{N\beta}\left(\widetilde{s}-\overline{\theta}\right)\text{ and }\widetilde{x}\left(s;0,\chi\right)=\frac{\alpha-\overline{\theta}}{\lambda+N\beta}-\frac{A(0,\chi)}{N\beta}\left(\widetilde{s}-\overline{\theta}\right).$$

In addition, we have shown in the proof of Corollary 4(ii) that the weight of the average signal on the price is lower with cursed sellers than with fully rational sellers, i.e., $A(0,0)>A(0,\chi)$. This implies that the aggregate quantity produced in the market ($N\widetilde{x}\left(s;0,\chi\right)$) is too high (low) with respect to the efficient outcome ($N\widetilde{x}\left(s;0,0\right)$) whenever $\widetilde{s}>\overline{\theta}$ ($\widetilde{s}<\overline{\theta}$). Moreover, increasing the degree of cursedness amplifies aggregate inefficiency in perfectly competitive markets.

With respect to distributive inefficiency in perfectly competitive markets, from equation (15) we obtain the following

$$u_j\left(s;0,0\right)=a(0,0)\left(\widetilde{s}-s_j\right)\text{ and }u_j\left(s;0,\chi\right)=a(0,\chi)\left(\widetilde{s}-s_j\right).$$

From Corollary 2, we derive that cursed sellers in equilibrium respond more to private information than fully rational sellers, i.e., $a(0,\chi)>a(0,0)$. Hence, in equilibrium suppliers whose private signals are higher (lower) than the average signal sell too little (too much) with respect to the efficient allocation. Therefore, we can conclude that in equilibrium sales are too disperse compared to the efficient allocation. Given that increasing the degree of cursedness increases the response to the private signal, which makes $a(0,\chi)$ further away from $a(0,0)$, we conclude that increasing the degree of cursedness raises the distortion related to the distribution of production for a given aggregate quantity, thus increasing distributive inefficiency.

To understand how imperfect competition changes the welfare analysis, we analyze aggregate inefficiency in the general model, which is proportional to

$$\mathbb{E}\left[\left(\widetilde{x}\left(s;0,0\right)-\widetilde{x}\left(s;d,\chi\right)\right)^2\right]=\left(\frac{\alpha-\overline{\theta}}{\lambda+N\beta}-\frac{\alpha-\overline{\theta}}{d+\lambda+N\beta}\right)^2+\frac{\left(A(0,0)-A(d,\chi)\right)^2}{N^2\beta^2}var\left[\widetilde{s}\right]. \quad (17)$$

The first term of (17) is affected by cursedness through the price impact. Hence, a higher degree of cursedness makes the expected equilibrium average quantity closer to the expected efficient average quantity, which reduces aggregate inefficiency. Moreover, with respect to the second term of (17), we find that it increases (decreases) with cursedness whenever $A(d,\chi)$ decreases (increases) with cursedness given that $A(0,0)>A(d,\chi)$. Hence, if $\alpha-\overline{\theta}$ is large enough, then the first term of (17) dominates and we can unambiguously conclude that aggregate inefficiency decreases with cursedness. This result also holds under the parameter configurations such that $A(d,\chi)$ increases with cursedness, as shown in Proposition A.1 in the Appendix. Otherwise, the opposite result may arise. By comparison, note that when sellers are fully rational as in Vives (2011), there is aggregate inefficiency at the equilibrium allocation due to strategic behavior. In our setup, aggregate inefficiency is also affected by the degree of cursedness, which can magnify or reduce this inefficiency in relation to a model with fully rational traders.

For distributive inefficiency, we show that it is proportional to $\mathbb{E}\left[\left(u_i\left(s;0,0\right)-u_i\left(s;d,\chi\right)\right)^2\right]$

for a given i, which is equal to

$$\mathbb{E}\left[\left(u_i\left(s;0,0\right)-u_i\left(s;d,\chi\right)\right)^2\right]=\left(a(0,0)-a(d,\chi)\right)^2 var\left[\widetilde{s}-s_i\right]. \tag{18}$$

Moreover, it can be shown that if ρ is low or χ is low, then $a(0,0)<a(d,\chi)$. Then, increasing the degree of cursedness enhances the sellers' response to private information ($a(d,\chi)$), moving it away from $a(0,0)$. This allows us to conclude that in this case distributive inefficiency increases with cursedness. Otherwise, i.e., if ρ and χ are high, then it holds that $a(0,0)>a(d,\chi)$. Now, $a(d,\chi)$ gets closer to $a(0,0)$ due to an increase of cursedness, and hence, distributive inefficiency decreases with cursedness. By comparison, note that when sellers are fully rational, Vives (2011) finds that sales are too similar among sellers, thus generating distributive inefficiency.

With a perfectly inelastic demand, only distributive inefficiency matters since there is no aggregate inefficiency at the equilibrium allocation. Then, combining this fact and Proposition 7(ii), we can conclude that if ρ is low or χ are low enough, then expected deadweight loss decreases with cursedness, as stated in Corollary 7(ii). This is because cursedness reduces the distortion in the distribution of production for a given aggregate quantity because sales become less similar among sellers.

5.2 Expected profits

One could think that when expected total surplus increases with cursedness is due to the fact that consumers are better off in a market where sellers are highly cursed. However, we show below that we can obtain the counter-intuitive result that sellers' expected profits can increase with cursedness.

To illustrate this result, we focus on the benchmark case of the perfectly inelastic demand. Then, sellers' expected profits, $\mathbb{E}[\pi_i]$, can be written as

$$\mathbb{E}[\pi_i]=\mathbb{E}[p-\theta_i-\frac{\lambda}{2}x_i]\mathbb{E}\left[x_i\right]+cov\left[p-\theta_i-\frac{\lambda}{2}x_i,x_i\right] \tag{19}$$

$$=\frac{2d+\lambda}{2}\frac{Q^2}{N^2}+cov\left[-\theta_i,x_i\right]-\frac{\lambda}{2}var\left[x_i\right] \tag{20}$$

Note that the first term of expected profits in equation (20) increases with price impact, and therefore, decreases with the degree of cursedness. Intuitively, increasing the degree of cursedness raises the expected quantity supplied and decreases the expected price, which results in a reduction in the expected profit per-unit. Hence, when cursedness increases, the expected profit per-unit decreases more than in proportion to the increase in the expected quantity, and this leads to the decrease in the price above average cost.

The second term of expected profits in equation (19) has an ambiguous sign with respect to cursedness because both $cov\left[-\theta_i,x_i\right]=acov\left[-\theta_i,\widetilde{s}-s_i\right]$, and the variance of the quantity supplied increase with cursedness. When ρ is small, although increasing cursedness raises the variance of the quantity supplied, this effect is dominated by the increase in the covariance term ($cov\left[-\theta_i,x_i\right]$,) which results in an increase of this part of expected profits. By contrast, when ρ is large the opposite result might hold. To illustrate this point, note that in the limiting case of common values ($\rho=1$), $cov\left[-\theta_i,\widetilde{s}-s_i\right]=cov\left[-\theta_i,\frac{\sum_j\varepsilon_j}{N}-\varepsilon_i\right]=0$, and hence,

$cov\left[-\theta_i, x_i\right] = 0$. This implies that

$$cov\left[p - \theta_i - \frac{\lambda}{2}x_i, x_i\right] = -\frac{\lambda}{2}var\left[x_i\right],$$

and therefore, we can conclude that the second term of $\mathbb{E}[\pi_i]$ in equation (19) decreases with cursedness.

The following proposition formalizes these results in the benchmark models.

Proposition 8: *Suppose that the demand is perfectly inelastic. Then, it holds that if Q is high enough, then sellers' expected profits decreases with cursedness. In contrast, if Q is low enough, three cases can be distinguished*:
(i) *when ρ is low enough, sellers' expected profits increase with cursedness;*
(ii) *when ρ is large enough* $($*but* $\rho < 1)$, *sellers' expected profits increase* $($*decrease*$)$ *with cursedness for low* $($*large*$)$ *values of* χ*; and*
(iii) *when* $\rho = 1$, *sellers' expected profits decrease for all values of* χ.

Our results show that, in the benchmark model of a perfectly inelastic demand, cursed sellers can earn higher profits than fully rational sellers when the demanded quantity is small and either the cost correlation is low or the cost correlation is high (except from common values) and the degree of cursedness is low. In these scenarios, cursed sellers can better align the quantity sold to the profit per unit, i.e., sellers supply a high (small) quantity when the profit per unit is high (low).

6 Concluding Remarks

In this paper we have studied a market with competition in schedules with boundedly rational sellers that partially neglect the informational content of the market price. We have used the cursed expectations equilibrium concept to analyze this setting. Our paper is the first to analyze the effects of cursedness in imperfectly competitive markets where sellers compete to supply an elastic demand. We have found that price impact is lower with cursed sellers than with fully rational sellers and that this has implications for market quality and welfare. Specifically, we find that in contrast to the main results of the literature, the variance of the market price (or the weight of the average signal on the price) may by higher and trading volume may be lower in markets with cursed traders compared to markets with fully rational sellers. In terms of welfare, we characterize the conditions under which total surplus may be higher in a market with cursed sellers compared to a market with fully rational ones. We also find that cursed sellers can better align the quantity sold to the profit per unit and this is the reason why cursed sellers might obtain higher profits than fully rational ones. Our results have various implications for policy. Our findings show that cursedness is not necessarily detrimental to either total surplus or to sellers' expected profits.

Future work could focus on markets comprised of asymmetries between sellers (such as, for example, the degree of cursedness), and on exploring how related behavioral biases interact with cursedness in a context of strategic behaviour and imperfect information.

References

[1] Angeletos, G. M., and Pavan, A. (2007). Efficient use of information and social value of information. Econometrica, 75(4), 1103-1142.

[2] Barberis, N., and Thaler, R. (2005). A survey of behavioral finance. Handbook of the Economics of Finance, 1, 1053-1128.

[3] Barberis, N. (2018). Psychology-based models of asset prices and trading volume. In Handbook of Behavioral Economics: Applications and Foundations 1 (Vol. 1, pp. 79-175). North-Holland.

[4] Bayona, A., Brandts, J., and Vives, X., (2020). Supply function competition, private information and market power: A laboratory study, Games and Economic Behavior, 122, 354-369.

[5] Benos, A. V. (1998). Aggressiveness and survival of overconfident traders. Journal of Financial Markets, 1(3-4), 353-383.

[6] Blume, L., and Easley, D. (1992). Evolution and market behavior. Journal of Economic theory, 58(1), 9-40.

[7] Bolle, F., Grimm, V., Ockenfels, A., and Del Pozo, X. (2013). An experiment on supply function competition. European Economic Review, 63, 170-185.

[8] Brandts, J., Reynolds, S. S., and Schram, A. (2014). Pivotal suppliers and market power in experimental supply function competition. The Economic Journal, 124(579), 887-916.

[9] Brown, D. P., and Eckert, A. (2021). Analyzing firm behavior in restructured electricity markets: Empirical challenges with a residual demand analysis. International Journal of Industrial Organization, 74, 102676.

[10] Camerer, C. F. (2003). Behavioral game theory: Experiments in strategic interaction. Princeton University Press.

[11] Charness, G., and Levin, D. (2009). The origin of the winner's curse: a laboratory study. American Economic Journal: Microeconomics, 1(1), 207-36.

[12] De Long, J. B., Shleifer, A., Summers, L. H., and Waldmann, R. J. (1990). Noise trader risk in financial markets. Journal of Political Economy, 98(4), 703-738.

[13] Esponda, I., and Vespa, E. (2014). Hypothetical thinking and information extraction in the laboratory. American Economic Journal: Microeconomics, 6(4), 180-202.

[14] Eyster, E., and Rabin, M. (2005). Cursed equilibrium. Econometrica, 73(5), 1623-1672.

[15] Eyster, E., Rabin, M., and Vayanos, D. (2019). Financial markets where traders neglect the informational content of prices. The Journal of Finance, 74(1), 371-399.

[16] Eyster, E. (2019). Errors in strategic reasoning. In Handbook of Behavioral Economics: Applications and Foundations 1 (Vol. 2, pp. 187-259). North-Holland.

[17] Grossman, S. J., and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. The American economic review, 70(3), 393-408.

[18] Grossman, S. J. (1981). An introduction to the theory of rational expectations under asymmetric information. The Review of Economic Studies, 48(4), 541-559.

[19] Hart, O. D. (1985). Monopolistic competition in the spirit of Chamberlin: A general model. The Review of Economic Studies, 52(4), 529-546.

[20] Hirshleifer, D., Subrahmanyam, A., and Titman, S. (2006). Feedback and the success of irrational investors. Journal of Financial Economics, 81(2), 311-338.

[21] Hirshleifer, D. (2015). Behavioral finance. Annual Review of Financial Economics, 7, 133-159.

[22] Holmberg, P., and Willems, B. (2015). Relaxing competition through speculation: Committing to a negative supply slope. Journal of Economic Theory, 159, 236-266.

[23] Hortacsu, A., and Puller, S. L. (2008). Understanding strategic bidding in multiunit auctions: a case study of the Texas electricity spot market. The RAND Journal of Economics, 39(1), 86-114.

[24] Kagel, J.H., and Levin, D., (2002). Common Value Auctions and the Winner's Curse. Princeton University Press, Princeton.

[25] Kyle, A. S., and Wang, F. A. (1997). Speculation duopoly with agreement to disagree: Can overconfidence survive the market test?. The Journal of Finance, 52(5), 2073-2090.

[26] Klemperer, P. D., and Meyer, M. A. (1989). Supply function equilibria in oligopoly under uncertainty. Econometrica: Journal of the Econometric Society, 1243-1277.

[27] Kyle, A. S. (1989). Informed speculation with imperfect competition. The Review of Economic Studies, 56(3), 317-355.

[28] Mondria, J., Vives, X., and Yang, L. (2021). Costly interpretation of asset prices. Management Science, 68(1): 52-74.

[29] Ngangoué, M. K., and Weizsäcker, G. (2021). Learning from unrealized versus realized prices. American Economic Journal: Microeconomics, 13(2): 174-201.

[30] Radner, R. (1962). Team decision problems. Annals of Mathematical Statistics 33 (3), 857–881.

[31] Rostek, M., and Weretka, M. (2012). Price inference in small markets. Econometrica, 80(2), 687-711.

[32] Spiegler, R. (2011). Bounded rationality and industrial organization. Oxford University Press.

[33] Tsagris, M., Beneki, C., and Hassani, H. (2014). On the folded normal distribution. Mathematics, 2(1), 12-28.

[34] Vives, X. (1988). Aggregation of information in large Cournot markets. Econometrica, 56, 851-876.

[35] Vives, X. (2011). Strategic supply function competition with private information. Econometrica, 79(6), 1919-1966.

[36] Wolak, F. (2015). Measuring the Competitiveness Benefits of a Transmission Investment Policy: The Case of the Alberta Electricity Market, Energy Policy, 85: 426-444.

Appendix

Proof of Proposition 1: From the distribution of random variables described in the model section, we obtain that

$$\begin{pmatrix} \theta_i \\ s_i \\ I_i \end{pmatrix} \sim N\left(\left(\begin{array}{c} \overline{\theta} \\ \overline{\theta} \\ \frac{\alpha-\beta(N-1)\left(b-a\overline{\theta}\right)}{1+\beta(N-1)c} \end{array}\right), \left(\begin{array}{ccc} \sigma_\theta^2 & \sigma_\theta^2 & \frac{\beta a(N-1)\rho\sigma_\theta^2}{1+\beta(N-1)c} \\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & \frac{\beta a(N-1)\rho\sigma_\theta^2}{1+\beta(N-1)c} \\ \frac{\beta a(N-1)\rho\sigma_\theta^2}{1+\beta(N-1)c} & \frac{\beta a(N-1)\rho\sigma_\theta^2}{1+\beta(N-1)c} & \frac{\beta^2 a^2(N-1)\left((1+(N-2)\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)}{(1+\beta(N-1)c)^2} \end{array}\right)\right).$$

Applying the standard normal theory, we know that

$$\mathbb{E}\left[\theta_i|s_i,I_i\right] = \overline{\theta} + \frac{\left((1-\rho)\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\sigma_\theta^2}{\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)}\left(s_i-\overline{\theta}\right) + \frac{\left(1+\beta\left(N-1\right)c\right)\rho\sigma_\theta^2\sigma_\varepsilon^2}{\beta\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)a}\left(I_i-\frac{\alpha-\beta(N-1)\left(b-a\overline{\theta}\right)}{1+\beta(N-1)c}\right) \text{ and}$$

$$\mathbb{E}\left[\theta_i|s_i\right] = \overline{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\left(s_i-\overline{\theta}\right).$$

From (2), we have $x_i = \frac{(1-\chi)(p-\mathbb{E}[\theta_i|s_i,I_i])+\chi(p-\mathbb{E}[\theta_i|s_i])}{d+\lambda}$. Substituting the previous expressions for $\mathbb{E}\left[\theta_i|s_i,I_i\right]$ and $\mathbb{E}\left[\theta_i|s_i\right]$, taking into account that $I_i = p + d\left(b - as_i + cp\right)$, in the resulting equation and identifying coefficients, it follows that

$$b = \frac{-(1-\chi)\frac{\frac{\left((1+\rho(N-2))\sigma_\theta^2+\sigma_\varepsilon^2\right)\sigma_\varepsilon^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}\overline{\theta}+\frac{(1+\beta(N-1)c)\rho\sigma_\theta^2\sigma_\varepsilon^2}{\beta\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)a}\left(db-\frac{\alpha-\beta(N-1)\left(b-a\overline{\theta}\right)}{1+\beta(N-1)c}\right)}{(1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2}-\chi\frac{\sigma_\varepsilon^2\overline{\theta}}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda},$$

$$a = \frac{(1-\chi)\frac{\frac{\left((1-\rho)(1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}+\frac{(1+\beta(N-1)c)\rho\sigma_\theta^2\sigma_\varepsilon^2}{\beta\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)}(-d)}{(1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2}+\chi\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}, \text{ and}$$

$$c = \frac{(1-\chi)\left(1-\frac{(1+\beta(N-1)c)\rho\sigma_\theta^2\sigma_\varepsilon^2}{\beta\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)a}(1+dc)\right)+\chi}{d+\lambda}.$$

Using the expression of d and doing some algebra, we obtain

$$b = \frac{(1-\chi)\frac{-\sigma_\varepsilon^2\overline{\theta}+\frac{\sigma_\varepsilon^2\sigma_\theta^2\rho(\alpha-Nb\beta)}{\beta a\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)}}{(1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2}-\chi\frac{\sigma_\varepsilon^2\overline{\theta}}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}, \tag{21}$$

$$a = \frac{(1-\chi)\frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}+\chi\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}, \text{ and} \tag{22}$$

$$c = \frac{1-\frac{(1-\chi)\rho\sigma_\theta^2\sigma_\varepsilon^2(1+N\beta c)}{\beta\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)a}}{d+\lambda}. \tag{23}$$

Substituting (22) into (21) and (23), and isolating b and c in the resulting expressions, we get (5) and (6).

In addition, using (6) in the expression of d and after some algebra, it follows that in equilibrium d is the solution of the following polynomial:

$$R(d) = d^2 (N + M) + d (\beta N (N - 2 - M) + \lambda (M + N)) - \lambda\beta N (1 + M).$$

Note that, since $\lambda > 0$, this polynomial has a positive and a negative root. Moreover, the unique root that is compatible with the second order condition is the highest one and it is given by

$$d = \frac{\beta N(M-N+2)-\lambda(M+N)+\sqrt{\beta^2 N^2 (M-(N-2))^2+\lambda^2 (N+M)^2+2\beta\lambda N(N+M)^2}}{2(N+M)}.$$

Finally, using the market clearing condition and the expressions of the linear and identical supply functions, we get

$$\sum\nolimits_{j=1}^{N} (b - as_j + cp) = \frac{\alpha - p}{\beta}, \tag{24}$$

which implies that equation (8) holds, with $A = \frac{N\beta a}{1+N\beta c}$. Finally, substituting (4), (6), and (7) into the previous expression for A, we obtain (9).

Proof of Corollary 1:
(i) Define $F(d,\rho) = R(d)$. Note that $\frac{\partial}{\partial\rho}(F(d,\chi)) = \frac{\partial}{\partial M}(R(d))\frac{\partial M}{\partial\rho}$

$$= \frac{(d+\lambda)(d-N\beta) N\sigma_\varepsilon^2 (1-\chi)\left(\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\left(1+\rho^2 (N-1)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)-\chi\rho^2\sigma_\theta^2\sigma_\varepsilon^2 (N-1)\right)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(1-\rho+\chi\frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)^2\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)^2} < 0$$

since in equilibrium $d + \lambda > 0$ and $d < N\beta$ and the fact that $\sigma_\varepsilon^2 (1-\chi) > 0$.[11] Moreover, the shape of the polynomial allows us to conclude that $\frac{\partial}{\partial d}(F(d,\chi)) > 0$. Applying the Implicit Function Theorem, we have $\frac{\partial d}{\partial\rho} > 0$, which implies that $\frac{\partial c}{\partial\rho} < 0$. In addition, it is easy to see that the numerator of a in (22) decreases in ρ and the denominator increases in ρ whenever $\sigma_\varepsilon^2 (1-\chi) > 0$. Consequently, $\frac{\partial a}{\partial\rho} < 0$.
(ii) Define $F(d,\chi) = R(d)$. Note that $\frac{\partial}{\partial\chi}(F(d,\chi)) = \frac{\partial}{\partial M}(R(d))\frac{\partial M}{\partial\chi}$

$$= (d+\lambda)(N\beta - d)\frac{\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\rho N\sigma_\varepsilon^2}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(1-\rho+\chi\frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)^2\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)} > 0$$

since in equilibrium $d+\lambda > 0$ and $d < N\beta$ and the fact that $\rho\sigma_\varepsilon^2 > 0$. Moreover, we know that $\frac{\partial}{\partial d}(F(d,\chi)) > 0$. Applying the Implicit Function Theorem, we have $\frac{\partial d}{\partial\chi} < 0$.

Using the expression of d, $\frac{\partial d}{\partial\chi} < 0$ implies that $\frac{\partial c}{\partial\chi} > 0$. In addition, it is easy to see that the numerator of a in (22) increases in χ and the denominator decreases in χ whenever $\rho\sigma_\varepsilon^2 > 0$. Consequently, $\frac{\partial a}{\partial\chi} > 0$.
(iii) Define $F(d,\beta) = R(d)$. Note that $\frac{\partial d}{\partial\beta} = -\frac{\frac{\partial}{\partial\beta}(F(d,\beta))}{\frac{\partial}{\partial d}(F(d,\beta))}$. Again, as $\frac{\partial}{\partial d}(F(d,\chi)) > 0$, we get $sign\left(\frac{\partial d}{\partial\beta}\right) = -sign\left(\frac{\partial}{\partial\beta}(F(d,\beta))\right)$. In addition,

$$\frac{\partial}{\partial\beta}(F(d,\beta)) = \frac{\partial}{\partial\beta}(R(d)) = -N(d(M-N+2)+\lambda(1+M)). \tag{25}$$

[11] Note that $R(N\beta) = \beta N(N-1)(\lambda + 2N\beta) > 0$, which implies that $N\beta$ is higher than the highest root of $R(d)$.

We distinguish two cases: 1) $M-N+2 \geq 0$, and 2) $M-N+2<0$.

Case 1: $M-N+2 \geq 0$. In this case, expression (25) allows us to conclude that $\frac{\partial}{\partial \beta}(F(d,\beta))<0$, which implies that $\frac{\partial d}{\partial \beta}>0$.

Case 2: $M-N+2<0$. Direct computations yield that $R\left(-\frac{\lambda(1+M)}{M-N+2}\right)=\frac{\lambda^2(N-1)(1+M)(N+M)}{(M-N+2)^2}>0$. Hence, it follows that $d<-\frac{\lambda(1+M)}{M-N+2}$, and therefore, expression (25) implies that $\frac{\partial}{\partial \beta}(F(d,\beta))<0$, which leads us to conclude that $\frac{\partial d}{\partial \beta}>0$.

In relation to the equilibrium coefficients a and c, from Proposition 1, we find that

$$sign\left(\frac{\partial a}{\partial \beta}\right)=-sign\left(\frac{\partial d}{\partial \beta}\right)=- \text{ and}$$

$$\frac{\partial c}{\partial \beta}=\frac{1}{1+M}\left(\frac{M}{N\beta^2}-\frac{\partial d}{\partial \beta}(d+\lambda)^{-2}\right).$$

Concerning the last derivative, given that $\frac{\partial d}{\partial \beta}=\frac{N(d(M-N+2)+\lambda(1+M))}{2d(N+M)+(\beta N(N-2-M)+\lambda(M+N))}$, it can be written as

$$\frac{\partial c}{\partial \beta}=\frac{-\beta^2N^2\left(d\left(M-N+2\right)+\lambda\left(1+M\right)\right)-\beta MN\left(d+\lambda\right)^2\left(M-N+2\right)+M\left(2d+\lambda\right)\left(d+\lambda\right)^2\left(M+N\right)}{N\beta^2\left(d+\lambda\right)^2\left(1+M\right)\left(2d\left(N+M\right)+\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right)\right)}.$$

In addition, as $R(d)=0$ implies that $N\beta\left(\lambda\left(1+M\right)+d\left(M-N+2\right)\right)=d\left(d+\lambda\right)\left(M+N\right)$, we have

$$\frac{\partial c}{\partial \beta}=\frac{2M(M+N)d^2+(3M\lambda(M+N)-N\beta(M+N)-MN\beta(M-N+2))d+\left(M\lambda^2(M+N)-MN\beta\lambda(M-N+2)\right)}{N\beta^2(d+\lambda)(1+M)(2d(N+M)+(\beta N(N-2-M)+\lambda(M+N)))}.$$

Again, $R(d)=0$ implies that $\left(M+N\right)d^2=-\left(d\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right)-\lambda\beta N\left(1+M\right)\right)$. Therefore,

$$\frac{\partial c}{\partial \beta}=\frac{\left(M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\right)d+M\lambda\left(\lambda+N\beta\right)\left(M+N\right)}{N\beta^2\left(d+\lambda\right)\left(1+M\right)\left(2d\left(N+M\right)+\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right)\right)}.$$

As the denominator of $\frac{\partial c}{\partial \beta}$ is positive, we conclude that

$$sign\left(\frac{\partial c}{\partial \beta}\right)=sign\left(\left(M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\right)d+M\lambda\left(\lambda+N\beta\right)\left(M+N\right)\right). \tag{26}$$

Next, we distinguish two cases: 1) $M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\geq 0$, and 2) $M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)<0$.

Case 1: $M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\geq 0$ (i.e., $M\geq N$ or $M<N$ and $\beta\leq\frac{M}{N}\frac{\lambda(M+N)}{(N-M)(1+M)}$). In this case (26) implies that $\frac{\partial c}{\partial \beta}>0$.

Case 2: $M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)<0$ (i.e., $M<N$ and $\beta>\frac{M}{N}\frac{\lambda(M+N)}{(N-M)(1+M)}$). In this case, from (26), we have that $\frac{\partial c}{\partial \beta}>0$ is equivalent to $d<\frac{M\lambda(\lambda+N\beta)(M+N)}{-(M\lambda(M+N)+N\beta(M-N)(1+M))}$, or $p\left(\frac{M\lambda(\lambda+N\beta)(M+N)}{-(M\lambda(M+N)+N\beta(M-N)(1+M))}\right)>0$. Direct computations yield

$$R\left(\frac{M\lambda\left(\lambda+N\beta\right)\left(M+N\right)}{-\left(M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\right)}\right)=\frac{N^2\beta\lambda\left(1+M\right)G(\beta)}{\left(-\left(M\lambda\left(M+N\right)+N\beta\left(M-N\right)\left(1+M\right)\right)\right)^2},$$

with

$$G(\beta)=\beta^2 N\left(N-M\right)\left(-NM^2+\left(N^2-4N+1\right)M-N\right)+2\beta MN\lambda\left(M+N\right)^2+M\lambda^2\left(M+N\right)^2.$$

Note that $G(\beta)$ is a polynomial of degree 2 in β. Let $\widetilde{N}=\frac{4M+M^2+1+(1+M)\sqrt{6M+M^2+1}}{2M}$. If $N\geq\widetilde{N}$, then all the coefficients of $G(\beta)$ are positive. Thus, $R\left(\frac{M\lambda(\lambda+N\beta)(M+N)}{-(M\lambda(M+N)+N\beta(M-N)(1+M))}\right)>0$, which implies $\frac{\partial c}{\partial\beta}>0$. Otherwise if $M<N<\widetilde{N}$. In this the coefficient associated with β^2 is negative, while the other coefficients are positive. Moreover, given that in this subcase $\beta>\frac{M}{N}\frac{\lambda(M+N)}{(N-M)(1+M)}$ and that

$$G\left(\frac{M}{N}\frac{\lambda\left(M+N\right)}{\left(N-M\right)\left(1+M\right)}\right)=\frac{M\lambda^2\left(-M+N+2MN\right)^2\left(M+N\right)^2}{N\left(1+M\right)^2\left(N-M\right)}>0 \text{ and } \underset{\beta\to\infty}{lim}G(\beta)=-\infty,$$

we can conclude that if β is low enough, then $G\left(\beta\right)>0$, which implies that $\frac{\partial c}{\partial\beta}>0$. Otherwise, $G\left(\beta\right)<0$, and therefore, $\frac{\partial c}{\partial\beta}<0$.

Proof of Corollary 2: This follows directly by substituting $d=0$ into the equilibrium equations of Proposition 1.

Proof of Corollary 3: From the proof of Proposition 1, we have that the value of d in equilibrium satisfies $R(d)=0$, which can be rewritten as

$$-N\left(\lambda\left(1+M\right)+d\left(M-N+2\right)\right)\beta+d\left(d+\lambda\right)\left(M+N\right)=0.$$

Dividing the previous expression by β, and taking the limit when β converges to infinity, it follows that

$$\lambda\left(1+M\right)+d\left(M-N+2\right)=0,$$

which implies that $d=\frac{\lambda(1+M)}{N-2-M}$. Taking into account the second order condition (which implies that $d+\lambda>0$), we conclude that the equilibrium exists provided that $N-2>M$.

Proof of Proposition 2: Using (3),

$$p=(1-\chi)\mathbb{E}\left[\theta_i|s_i,p\right]+\chi\mathbb{E}\left[\theta_i|s_i\right]+\left(d+\lambda\right)x_i.$$

Hence, the difference between the market price and the seller i's marginal cost, $p-\left(\theta_i+\lambda x_i\right)$, is given by

$$(1-\chi)\mathbb{E}\left[\theta_i|s_i,p\right]+\chi\mathbb{E}\left[\theta_i|s_i\right]+dx_i-\theta_i.$$

Taking expectations in the previous expression, we get $d\mathbb{E}\left[x_i\right]$.

Furthermore, note that $\mathbb{E}\left[x_i\right]=b-a\overline{\theta}+c\mathbb{E}\left[p\right]$. Isolating p from (24), the previous expression implies that

$$\mathbb{E}\left[x_i\right]=\frac{b+c\alpha-a\overline{\theta}}{1+\beta Nc}.$$

Using the expressions given in Proposition 1, we get

$$\mathbb{E}[x_i] = \frac{\alpha - \overline{\theta}}{(1+\beta Nc)(d+\lambda)(1+M)}.$$

Moreover, from (6), we have $(1+\beta Nc)(d+\lambda)(1+M) = d+\lambda+N\beta$. Hence,

$$\mathbb{E}[x_i] = \frac{\alpha - \overline{\theta}}{d+\lambda+N\beta}. \tag{27}$$

Thus, the expected price-cost margin is given by $d\frac{\alpha-\overline{\theta}}{d+\lambda+N\beta}$. Differentiating with respect to χ, we get

$$\frac{\partial}{\partial\chi}(d\mathbb{E}[x_i]) = \left(\alpha - \overline{\theta}\right)\frac{\lambda+N\beta}{(d+\lambda+N\beta)^2}\frac{\partial d}{\partial\chi} < 0,$$

since $\alpha > \overline{\theta}$ and $\frac{\partial d}{\partial\chi} < 0$.

The following result is useful to prove Proposition 3.

Proposition A.1: *Suppose that $\rho\sigma_\varepsilon^2 > 0$ and $\lambda > 0$. In the equilibrium, it holds that*

1. *if $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \geq \frac{N^2\beta}{\lambda}$, then A decreases with cursedness.*
2. *if $\frac{1}{\beta\lambda}\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2 < \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \frac{N^2\beta}{\lambda}$, then there exists a value of ρ, denoted by $\widehat{\rho}$ (which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda}$) such that*
   - *if $\rho \leq \widehat{\rho}$, then A decreases with cursedness.*
   - *if $\rho > \widehat{\rho}$, then there exists a value of χ, denoted by $\widehat{\chi}$, with $0 < \widehat{\chi} < 1$ such that A decreases with cursedness if and only if $\chi > \widehat{\chi}$.*
3. *if $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \leq \frac{1}{\beta\lambda}\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2$, then there exists a value $\widehat{\widehat{\rho}}$ (which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=1}\right)^2}{\beta\lambda}$) such that*
   - *if $\rho \leq \widehat{\rho}$, then A decreases with cursedness.*
   - *if $\widehat{\rho} < \rho < \widehat{\widehat{\rho}}$, then there exists a value of χ, denoted by $\widehat{\chi}$, with $0 < \widehat{\chi} < 1$ such that A decreases with cursedness if and only if $\chi > \widehat{\chi}$.*
   - *if $\rho \geq \widehat{\widehat{\rho}}$, then A increases with cursedness.*

*Proof of Proposition A.*1: Differentiating the expression of A given in (9) with respect to χ, we get

$$\frac{\partial A}{\partial\chi} = -\frac{\frac{\rho\sigma_\theta^2\sigma_\varepsilon^2(N-1)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}}{\frac{d+\lambda}{N\beta}+1} - \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}+(1-\chi)\frac{\rho\sigma_\theta^2\sigma_\varepsilon^2(N-1)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}}{N\beta\left(\frac{d+\lambda}{N\beta}+1\right)^2}\frac{\partial d}{\partial\chi}.$$

Note that $\frac{\partial d}{\partial \chi} = \frac{\partial d}{\partial M}\frac{\partial M}{\partial \chi}$. Moreover, after some algebra, we have

$$\frac{\partial M}{\partial \chi} = -\frac{N\rho\sigma_{\varepsilon}^{2}\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\left(1-\rho\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)}{\left(\left(1+\rho\left(N-1\right)\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\left(1-\rho\right)\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)+\chi\rho\sigma_{\varepsilon}^{2}\right)^{2}} \text{ and} \tag{28}$$

$$\frac{\partial d}{\partial M} = \frac{\left(d+\lambda\right)\left(N\beta-d\right)}{2d\left(M+N\right)+\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right)}. \tag{29}$$

Combining these expressions, it follows that

$$\frac{\partial A}{\partial \chi} = -\frac{\frac{\rho\sigma_{\theta}^{2}\sigma_{\varepsilon}^{2}(N-1)}{\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left((1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)}}{\frac{d+\lambda}{N\beta}+1} + \\ \frac{\rho\sigma_{\theta}^{2}\sigma_{\varepsilon}^{2}\left((1-\rho)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left((1+\rho(N-1))\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)-\chi\rho\sigma_{\varepsilon}^{2}(N-1)\right)}{\left((1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)^{2}\left((1-\rho)\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)+\chi\rho\sigma_{\varepsilon}^{2}\right)^{2}}\frac{(d+\lambda)(N\beta-d)}{\beta\left(\frac{d+\lambda}{N\beta}+1\right)^{2}(2d(M+N)+(\beta N(N-2-M)+\lambda(M+N)))}.$$

Taking into account that

$$\left(d+\lambda\right)\left(N\beta-d\right) = \frac{d\left(N-1\right)\left(d+\lambda+N\beta\right)}{1+M}$$

and that

$$\chi = \frac{\frac{M}{N\rho\sigma_{\varepsilon}^{2}}\left(\rho-1\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)+1}{\frac{M}{N}\frac{(1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}}{\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}}+1}, \tag{30}$$

from expression (7), the previous expression can be rewritten as

$$\frac{\partial A}{\partial \chi} = \frac{N(N-1)\beta\rho\sigma_{\theta}^{2}\sigma_{\varepsilon}^{2}\left(\left((1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)M+N\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\right)}{\left((1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left((1-\rho)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)(d+\lambda+N\beta)} \times \\ \left(-\frac{(1-\rho)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}}{\left((1+\rho(N-1))\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)M+N\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)} + \frac{d}{2d(M+N)+(\beta N(N-2-M)+\lambda(M+N))}\right).$$

Note that the last fraction in the previous expression holds

$$\frac{d}{2d(M+N)+(\beta N(N-2-M)+\lambda(M+N))} = \frac{d^{2}}{2d^{2}(M+N)+d(\beta N(N-2-M)+\lambda(M+N))}.$$

In addition, using the fact that d is a root of the polynomial $R(d)$, the denominator of the RHS of the previous equation holds

$$2d^{2}\left(M+N\right)+d\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right) = d^{2}\left(M+N\right)+N\left(1+M\right)\beta\lambda.$$

Hence,

$$\frac{d}{2d\left(M+N\right)+\left(\beta N\left(N-2-M\right)+\lambda\left(M+N\right)\right)} = \frac{d^{2}}{\left(M+N\right)d^{2}+N\left(1+M\right)\beta\lambda},$$

and therefore,

$$\frac{\partial A}{\partial \chi} = \frac{N^{2}\beta\rho^{2}\sigma_{\theta}^{4}\sigma_{\varepsilon}^{2}\left(1+M\right)\left(N-1\right)\beta\lambda H(\chi)}{\left(\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\left(1-\rho\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}\right)\left(\left(M+N\right)d^{2}+N\left(1+M\right)\beta\lambda\right)\left(d+\lambda+N\beta\right)},$$

with[12]

$$H(\chi) = \frac{d^{2}}{\beta\lambda} - \frac{\left(1-\rho\right)\sigma_{\theta}^{2}+\sigma_{\varepsilon}^{2}}{\rho\sigma_{\theta}^{2}}.$$

[12] Remember that d is a function of χ. This is the reason why we can write H as a function of χ.

Consequently,

$$sign\left(\frac{\partial A}{\partial \chi}\right) = sign\left(H(\chi)\right). \tag{31}$$

Using Corollary 1, we know that d decreases in χ. Therefore, $H(\chi)$ decreases in χ. Then, we distinguish three cases:

Case 1: $\lim_{\chi\to 1} H(\chi) \geq 0$ $\left(\text{i.e., } \frac{\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2}{\beta\lambda} \geq \frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2}\right)$. In this case, from (31), we conclude that A increases with cursedness for all χ.

Case 2: $H(0) \leq 0$ $\left(\text{i.e., } \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda} \leq \frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2}\right)$. In this case, from (31), we conclude that A decreases with cursedness for all χ.

Case 3: $\lim_{\chi\to 1} H(\chi) < 0$ and $H(0) > 0$. In this case we have that there exists a value of χ, denoted by $\widehat{\chi}$, with $0 < \widehat{\chi} < 1$, such that A increases with cursedness if and only if $\chi < \widehat{\chi}$.

Let us define $G(\rho) = \lim_{\chi\to 1} H(\chi)$. Note that $G(\rho)$ is an increasing function in ρ and that $\lim_{\rho\to 0} G(\rho) = -\infty$ and $\lim_{\rho\to 1} G(\rho) = \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda} - \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$. Then, we distinguish two cases: $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} > \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \leq \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$.

Case 1: $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} > \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$. In this $\lim_{\chi\to 1} H(\chi) < 0$ for all ρ. Let us define $F(\rho) = H(0) = \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda} - \frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2}$. This function increases in ρ. Then, we distinguish two subcases: $F(1) \leq 0$ and $F(1) > 0$.

Subcase 1.1: $F(1) \leq 0$, i.e., $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \geq \lim_{\rho\to 1} \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda} = \frac{N^2\beta}{\lambda}$. In this case $H(0) \leq 0$ for all ρ, and hence, we conclude that A decreases with cursedness for all χ.

Subcase 1.2: $F(1) > 0$, i.e., $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \lim_{\rho\to 1} \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda} = \frac{N^2\beta}{\lambda}$. In this case there exists a value of ρ, denoted by $\widehat{\rho}$ (which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda}$) such that $H(0) > 0$ if and only if $\rho > \widehat{\rho}$. Therefore, whenever $\frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda} < \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \frac{N^2\beta}{\lambda}$, then if $\rho \leq \widehat{\rho}$, then $H(0) \leq 0$, which implies that A decreases with cursedness for all χ. Otherwise, i.e., $\rho > \widehat{\rho}$, given that in this case $\lim_{\chi\to 1} H(\chi) < 0$ and $H(0) > 0$, then we have that there exists a value of χ, denoted by $\widehat{\chi}$, with $0 < \widehat{\chi} < 1$, such that A decreases with cursedness if $\chi > \widehat{\chi}$.

Case 2: $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \leq \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$. In this case there exists a value of ρ, denoted by $\widehat{\widehat{\rho}}$, which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$, such that $\lim_{\chi\to 1} H(\chi) < 0$ if and only if $\rho < \widehat{\widehat{\rho}}$. Therefore, if $\rho \geq \widehat{\widehat{\rho}}$, then $\lim_{\chi\to 1} H(\chi) \geq 0$, and hence, we have that A increases with cursedness for all χ. Otherwise, i.e., if $\rho < \widehat{\widehat{\rho}}$, $\lim_{\chi\to 1} H(\chi) < 0$, then, we distinguish two cases: if $\rho \leq \widehat{\rho}$, then $H(0) < 0$, which implies that A decreases with cursedness for all χ. Otherwise, i.e., $\rho > \widehat{\rho}$, given that in this case $\lim_{\chi\to 1} H(\chi) < 0$

and $H(0) > 0$, then we have that there exists a value of χ, denoted by $\widehat{\chi}$, with $0 < \widehat{\chi} < 1$, such that A decreases with cursedness if and only if $\chi > \widehat{\chi}$.

To summarize this part, we have 3 possibilities:

- $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \geq \frac{N^2\beta}{\lambda}$. In this case A decreases with cursedness for all χ.
- $\frac{1}{\beta\lambda}\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2 < \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < \frac{N^2\beta}{\lambda}$. In this case there exists a value of ρ, denoted by $\widehat{\rho}$ (which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(d|_{\chi=0}\right)^2}{\beta\lambda}$) such that
  - if $\rho \leq \widehat{\rho}$, then A decreases with cursedness for all χ; and
  - if $\rho > \widehat{\rho}$, then there exists a value of χ, denoted by $\widehat{\chi}$, such that A decreases with cursedness if and only if $\chi > \widehat{\chi}$.
- $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \leq \frac{1}{\beta\lambda}\left(\frac{-\beta(N-2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2}\right)^2$. In this case there exists a value $\widehat{\widehat{\rho}}$, which is the solution of $\frac{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}{\rho\sigma_\theta^2} = \frac{\left(\lim_{\chi\to 1} d\right)^2}{\beta\lambda}$, such that
  - if $\rho \leq \widehat{\rho}$, then A decreases with cursedness for all χ;
  - if $\widehat{\rho} < \rho < \widehat{\widehat{\rho}}$, then there exists a value of χ, denoted by $\widehat{\chi}$, such that A decreases with cursedness if and only if $\chi > \widehat{\chi}$; and
  - if $\rho \geq \widehat{\widehat{\rho}}$, then A increases with cursedness for all χ.

Proof of Proposition 3:
(i) Recall that in equilibrium $p = \alpha - b\sum_{i=1}^N x_i$. Taking expectations in the previous expression and using (27), equation (11) holds. Differentiating this expression with respect to χ, we have

$$\frac{\partial \mathbb{E}[p]}{\partial \chi} = \frac{N\beta\left(\alpha - \overline{\theta}\right)}{(d+\lambda+N\beta)^2}\frac{\partial d}{\partial \chi}.$$

Given that $\alpha > \overline{\theta}$ and $\frac{\partial d}{\partial \chi} < 0$, we get $\frac{\partial \mathbb{E}[p]}{\partial \chi} < 0$.

(ii) In relation to the variance of prices, combining expression (12) and Proposition A.1, we obtain the results stated in this proposition.

Proof of Corollary 4: With perfect competition, substituting $d = 0$ in (9), it follows that

$$A = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2} + (1-\chi)\frac{\rho\sigma_\theta^2\sigma_\varepsilon^2(N-1)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}}{\frac{\lambda}{N\beta}+1},$$

which decreases with χ. Combining this fact and expression (12), we conclude that in this case *var* $[p]$ decreases with cursedness.

With a perfectly inelastic demand, to compute the coefficient A, we take the limit in (9) when $\beta \to \infty$, and it holds

$$A = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} + (1-\chi) \frac{\rho \sigma_\theta^2 \sigma_\varepsilon^2 (N-1)}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2 + \sigma_\varepsilon^2\right)}.$$

which decreases with χ. Again, this allows us to conclude that $var\,[p]$ decreases with cursedness.

Proof of Lemma 1:

(i) Using expression (27), it follows that $\frac{\partial \mathbb{E}[x_i]}{\partial \chi} = -\frac{\alpha - \overline{\theta}}{(d+\lambda+N\beta)^2} \frac{\partial d}{\partial \chi} > 0$ since $\alpha > \overline{\theta}$ and $\frac{\partial d}{\partial \chi} < 0$ because of Corollary 1(ii).

(ii) Using equation (13), we obtain

$$var[x_i] = a^2 var\left[(\widetilde{s} - s_i) - \frac{1}{1+N\beta c}\widetilde{s}\right].$$

Given that $cov\,[\widetilde{s} - s_i, \widetilde{s}] = 0$, we have

$$var[x_i] = a^2 \left(var\,[\widetilde{s} - s_i] + \frac{1}{(1+N\beta c)^2} var\,[\widetilde{s}]\right). \tag{32}$$

Plugging expression (30) into (4), we have

$$a = \frac{N\sigma_\theta^2}{(d+\lambda)\left(\left((1+\rho(N-1))\sigma_\theta^2 + \sigma_\varepsilon^2\right)M + N\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\right)}.$$

Using the previous expression for a, (6) and the following two expressions:

$$var\,[\widetilde{s} - s_i] = (N-1)\frac{(1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2}{N} \text{ and} \tag{33}$$

$$var\,[\widetilde{s}] = \frac{(1+\rho(N-1))\sigma_\theta^2 + \sigma_\varepsilon^2}{N}, \tag{34}$$

equation (32) can be rewritten as

$$var[x_i] = \left(\frac{N\sigma_\theta^2}{(d+\lambda)\left(\left((1+\rho(N-1))\sigma_\theta^2 + \sigma_\varepsilon^2\right)M + N\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\right)}\right)^2 \times$$
$$\left((N-1)\frac{(1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2}{N} + \frac{(1+M)^2(d+\lambda)^2}{(d+\lambda+N\beta)^2}\frac{(1+\rho(N-1))\sigma_\theta^2 + \sigma_\varepsilon^2}{N}\right).$$

Differentiating,

$$\frac{\partial var\,[x_i]}{\partial \chi} = \left(\frac{\partial var\,[x_i]}{\partial M} + \frac{\partial var\,[x_i]}{\partial d}\frac{\partial d}{\partial M}\right)\frac{\partial M}{\partial \chi}.$$

Recall that equation (28) implies that $\frac{\partial M}{\partial \chi} < 0$ whenever $\rho\sigma_\varepsilon^2 > 0$. Therefore,

$$sign\left(\frac{\partial var\,[x_i]}{\partial \chi}\right) = -sign\left(\frac{\partial var\,[x_i]}{\partial M} + \frac{\partial var\,[x_i]}{\partial d}\frac{\partial d}{\partial M}\right).$$

Direct computations yield

$\frac{\partial var[x_i]}{\partial d} = \frac{-2N\sigma_\theta^4}{\left(\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)M+N\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\right)^2}\left(\frac{(N-1)\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)}{(d+\lambda)^3}+\frac{(1+M)^2\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}{(d+\lambda+N\beta)^3}\right) < 0$ and $\frac{\partial d}{\partial M} > 0$ because of (29). Therefore, $\frac{\partial var[x_i]}{\partial d}\frac{\partial d}{\partial M} < 0$. In addition,

$$\frac{\partial var\left[x_i\right]}{\partial M} = \frac{2N\sigma_\theta^4\left(N-1\right)\left(\sigma_\theta^2-\rho\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)q(d+\lambda)}{\left(d+\lambda\right)^2\left(d+\lambda+N\beta\right)^2\left(\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)M+N\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\right)^3},$$

with

$$q(z) = Mz^2 - N\beta\left(2z+N\beta\right).$$

Note that $q(z)$ has a minimum at $z = \frac{N\beta}{M}$. Moreover, $q(z) < 0$ for all $0 \le z < \frac{N\beta}{M}$. Thus, when $d+\lambda < \frac{N\beta}{M}$, it follows that $\frac{\partial var[x_i]}{\partial M} < 0$, which implies that $\frac{\partial var[x_i]}{\partial \chi} > 0$.

In addition, let $\widetilde{z}$ represent the positive root of $q(z)$. Note that $\widetilde{z}$ is a decreasing function in M. In addition, $\lim_{M\to 0}\widetilde{z} = \infty$ and $\lim_{M\to\infty}\widetilde{z} = 0$.[13] By contrast, $d+\lambda$ is an increasing function in M and $\lim_{M\to\infty} d+\lambda = N\beta+\lambda$. This implies that there exists a unique value of M, denote by $\widetilde{M}$, such that for all $M < \widetilde{M}$ we have that $d+\lambda < \widetilde{z}$ and, hence, $q(d+\lambda) < 0$ and, therefore, $\frac{\partial var[x_i]}{\partial M} < 0$. By contrast, for all $M > \widetilde{M}$, it follows that $d+\lambda > \widetilde{z}$ and, hence, $q(d+\lambda) > 0$ and, therefore, $\frac{\partial var[x_i]}{\partial M} > 0$.

To sum up, we have that for all $M < \widetilde{M}$, $\frac{\partial var[x_i]}{\partial \chi} > 0$. By contrast, for all $M > \widetilde{M}$, $\frac{\partial var[x_i]}{\partial M} > 0$ and $\frac{\partial var[x_i]}{\partial d}\frac{\partial d}{\partial M} < 0$ and, therefore, in principle, the sign of $\frac{\partial var[x_i]}{\partial \chi}$ is ambiguous.

Proof of Corollary 5: Suppose that the demand is perfectly inelastic. In this case the market clearing condition implies that

$$Nb - a\sum_{j=1}^{N} s_j + cNp = Q,$$

and hence, $p = \frac{Q-Nb+a\sum_{j=1}^{N} s_j}{cN}$. Therefore, the quantity supplied can be rewritten as

$$x_i = b - as_i + cp = b - as_i + c\frac{Q-Nb+a\sum_{j=1}^{N} s_j}{cN} = \frac{Q}{N} + a\left(\widetilde{s}-s_i\right).$$

Hence,

$$var\left[x_i\right] = a^2 var\left[\widetilde{s}-s_i\right].$$

Differentiating this expression, we get $\frac{\partial var[x_i]}{\partial \chi} = 2a\frac{\partial a}{\partial \chi}var\left[\widetilde{s}-s_i\right] > 0$ whenever $\rho\sigma_\varepsilon^2 > 0$ since in this case it also holds that $\frac{\partial a}{\partial \chi} > 0$.

Proof of Proposition 4: Recall that given a normally distributed random variable z with mean μ and variance σ^2, the random variable $|z|$ has a folded normal distribution. It is well-known (e.g., Tsagris et al., 2014) that

$$\mathbb{E}\left[|z|\right] = \sqrt{\frac{2}{\pi}}\sigma e^{-\frac{\mu^2}{2\sigma^2}} + \mu\left(1-2\Phi\left(\frac{-\mu}{\sigma}\right)\right),$$

where $\Phi\left(.\right)$ represents the standard normal cumulative distribution function. This formula can

[13] M converges to infinity when $\chi = 0$ and ρ converges to 1.

be applied to compute trading volume, $\mathbb{E}[|x_i|]$, and be rewritten as

$$\mathbb{E}[|x_i|] = \sqrt{\frac{2}{\pi}} var[x_i]\, e^{-\frac{\mathbb{E}[x_i]^2}{2var[x_i]^2}} + \frac{2\mathbb{E}[x_i]}{\sqrt{\pi}} \left(\int_{\frac{-\mathbb{E}[x_i]}{\sqrt{2var[x_i]}}}^{0} e^{-y^2} dy \right), \tag{35}$$

where $\mathbb{E}[x_i]$ and $var[x_i]$ are the mean and variance of x_i. Differentiating (35) with respect to χ, we have

$$\begin{aligned}\frac{\partial \mathbb{E}[|x_i|]}{\partial \chi} &= \frac{2}{\sqrt{\pi}} \left(\int_{\frac{-\mathbb{E}[x_i]}{\sqrt{2}(var[x_i])^{1/2}}}^{0} e^{-y^2} dy \right) \frac{\partial \mathbb{E}[x_i]}{\partial \chi} + \sqrt{\frac{2}{\pi}} e^{-\frac{(\mathbb{E}[x_i])^2}{2var[x_i]}} \frac{\partial}{\partial \chi} (var[x_i])^{1/2} = \\ &= \frac{2}{\sqrt{\pi}} \left(\int_{\frac{-\mathbb{E}[x_i]}{\sqrt{2}(var[x_i])^{1/2}}}^{0} e^{-y^2} dy \right) \frac{\partial \mathbb{E}[x_i]}{\partial \chi} + \sqrt{\frac{2}{\pi}} e^{-\frac{(\mathbb{E}[x_i])^2}{2var[x_i]}} \frac{1}{2} (var[x_i])^{-1/2} \frac{\partial var[x_i]}{\partial \chi}.\end{aligned}$$

From Lemma 1, we know that $\frac{\partial \mathbb{E}[|x_i|]}{\partial \chi} > 0$ and that $\frac{\partial var[x_i]}{\partial \chi} > 0$ whenever $c > 0$. Using the expressions of these partial derivatives, we conclude that if $\alpha - \overline{\theta}$ is high enough, then $\frac{\partial \mathbb{E}[|x_i|]}{\partial \chi}$. By contrast, if $\alpha - \overline{\theta}$ is low enough, then $sign\left(\frac{\partial \mathbb{E}[|x_i|]}{\partial \chi}\right) = sign\left(\frac{\partial var[x_i]}{\partial \chi}\right)$, which is positive whenever $M < \widetilde{M}$.

Proof of Corollary 6 [*Trading volume with a perfectly inelastic demand*]: This proof directly follows from the combination of the proof of Proposition 4 and Corollary 5.

Proof of Proposition 5: From the equilibrium price equation (8), we know that the market price is a linear function of $\widetilde{s}$. Hence,

$$\tau = (var[\theta_i | \widetilde{s}])^{-1} \text{ and } \psi = \frac{var[\theta_i|s_i] - var[\theta_i | s_i, \widetilde{s}]}{var[\theta_i|s_i]},$$

which allows us to conclude that both τ and ψ do not depend on χ.

Proof of Proposition 6:
(i) In the linear-quadratic specification of the model, it follows that

$$TS(x) = \sum_{i=1}^{N} (\alpha - \theta_i) x_i - \frac{1}{2} \left(\beta \left(\sum_{i=1}^{N} x_i \right)^2 + \lambda \sum_{i=1}^{N} x_i^2 \right).$$

At the team-efficient solution, expected total surplus $\mathbb{E}[TS]$ is maximized under the constraint that firms use decentralized linear production strategies contingent on endogenous public information (the price p or the equivalent variable $\widetilde{s}$). That is,

$$\underset{\substack{\widehat{a},\widehat{b},\widehat{c} \\ x_i = \widehat{b} - \widehat{a} s_i + \widehat{c}\widetilde{s},\ i=1,\dots,N}}{max} \mathbb{E}[TS]$$

We use the Substitution Method to solve the previous optimization problem. Plugging the expressions of x_i, $i = 1, ..., N$, given in the constraints into the original objective function, we obtain an unconstrained problem with the following function, denoted by $F(\widehat{a},\widehat{b},\widehat{c})$, to be maximized:

$$F\left(\widehat{a},\widehat{b},\widehat{c}\right)=-\frac{N\left(\lambda\left(\overline{\theta}^2+\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\Psi\right)}{2}\widehat{a}^2-\frac{N(\lambda+N\beta)}{2}\widehat{b}^2-\frac{\Psi(\lambda+N\beta)}{2}\widehat{c}^2+N\overline{\theta}\left(\lambda+N\beta\right)\widehat{a}\widehat{b}+\Psi\left(\lambda+N\beta\right)\widehat{a}\widehat{c}$$
$$-N\overline{\theta}\left(\lambda+N\beta\right)\widehat{b}\widehat{c}+N\left(\sigma_\theta^2+\overline{\theta}\left(\overline{\theta}-\alpha\right)\right)\widehat{a}-N\left(\overline{\theta}-\alpha\right)\widehat{b}-\left(N\overline{\theta}\left(\overline{\theta}-\alpha\right)+\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2\right)\widehat{c},$$

with $\Psi=N\overline{\theta}^2+\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2$.

The first order conditions of this new optimization problem is given by:

$$\frac{\partial}{\partial\widehat{a}}F(\widehat{a},\widehat{b},\widehat{c})=0,$$
$$\frac{\partial}{\partial\widehat{b}}F(\widehat{a},\widehat{b},\widehat{c})=0,\ \text{and}$$
$$\frac{\partial}{\partial\widehat{c}}F(\widehat{a},\widehat{b},\widehat{c})=0.$$

Direct computations yield

$$-N\left(\lambda\left(\overline{\theta}^2+\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\Psi\right)\widehat{a}+\left(\lambda+N\beta\right)\left(N\overline{\theta}\widehat{b}+\Psi\widehat{c}\right)+N\left(\sigma_\theta^2+\overline{\theta}\left(\overline{\theta}-\alpha\right)\right)=0, \tag{36}$$

$$N\left(\lambda+N\beta\right)\left(-\widehat{b}+\overline{\theta}\widehat{a}-\overline{\theta}\widehat{c}\right)-N\left(\overline{\theta}-\alpha\right)=0,\ \text{and} \tag{37}$$

$$\left(\lambda+N\beta\right)\left(\Psi\left(\widehat{a}-\widehat{c}\right)-N\overline{\theta}\widehat{b}\right)-\left(N\overline{\theta}\left(\overline{\theta}-\alpha\right)+\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2\right)=0. \tag{38}$$

Isolating $\widehat{b}$ from (37), it follows that

$$\widehat{b}=\frac{\alpha-\overline{\theta}+\overline{\theta}\left(\lambda+N\beta\right)\widehat{a}-\overline{\theta}\left(\lambda+N\beta\right)\widehat{c}}{\lambda+N\beta}. \tag{39}$$

Substituting the previous expression into (36) and (38) and using the expression of Ψ, we get

$$-N\left(\lambda\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\right)\widehat{a}$$
$$+\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\lambda+N\beta\right)\widehat{c}+N\sigma_\theta^2=0\ \text{and} \tag{40}$$

$$-\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\lambda+N\beta\right)\widehat{c}+$$
$$\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\lambda+N\beta\right)\widehat{a}-\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2=0. \tag{41}$$

Isolating $\widehat{a}$ from (40), we have

$$\widehat{a}=\frac{\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\lambda+N\beta\right)\widehat{c}+N\sigma_\theta^2}{N\left(\lambda\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\right)}. \tag{42}$$

Substituting the above expression into (41) and isolating $\widehat{c}$ from the resulting expression, it follows that

$$\widehat{c}=N\sigma_\theta^2\frac{\beta\left(1-\rho\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)-\lambda\rho\sigma_\varepsilon^2}{\lambda\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\left(1+\rho\left(N-1\right)\right)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\lambda+N\beta\right)}. \tag{43}$$

Plugging (43) into (42), it follows that

$$\widehat{a} = \frac{(1-\rho)\,\sigma_\theta^2}{\lambda\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)}. \tag{44}$$

Substituting the previous two expressions into (39), we have

$$\widehat{b} = \frac{\alpha\,(1+\rho\,(N-1))\,\sigma_\theta^2+\left(\alpha-\overline{\theta}\right)\sigma_\varepsilon^2}{(\lambda+N\beta)\left((1+\rho\,(N-1))\,\sigma_\theta^2+\sigma_\varepsilon^2\right)}. \tag{45}$$

Now we apply a sufficient second order condition to show that the critical point is a maximum. The Hessian matrix of the function F is given by

$$HF(\widehat{a},\widehat{b},\widehat{c}) = \begin{bmatrix} -N\left(\lambda\left(\overline{\theta}^2+\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\Psi\right) & (\lambda+N\beta)\,N\overline{\theta} & \Psi\,(\lambda+N\beta) \\ (\lambda+N\beta)\,N\overline{\theta} & -N\,(\lambda+N\beta) & -(\lambda+N\beta)\,N\overline{\theta} \\ \Psi\,(\lambda+N\beta) & -(\lambda+N\beta)\,N\overline{\theta} & -\Psi\,(\lambda+N\beta) \end{bmatrix}.$$

The leading principal minors of this matrix satisfy

$\Delta_1 = -N\left(\lambda\left(\overline{\theta}^2+\sigma_\theta^2+\sigma_\varepsilon^2\right)+\beta\Psi\right) < 0,$
$\Delta_2 = N^2\,(\lambda+N\beta)\left(\left((1+\rho\,(N-1))\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\beta+\lambda\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\right) > 0$, and
$\Delta_3 = -N\,(N-1)\,\lambda\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho\,(N-1))\,\sigma_\theta^2+\sigma_\varepsilon^2\right)(\lambda+N\beta)^2 < 0$ since $\lambda > 0$.

Thus, $HF(\widehat{a},\widehat{b},\widehat{c})$ is negative definite and F is maximized at $\widehat{a}^*$, $\widehat{b}^*$, and $\widehat{c}^*$, whose expressions are given by (44), (45), and (43), respectively.

To end this part of the proof of Proposition 6, note that taking into account (15), direct computations yield that

$$x_i\,(s;0,0) = \widehat{b}^* - \widehat{a}^* s_i + \widehat{c}^*\widetilde{s},$$

which implies that the efficient allocation is $x\,(s;0,0)$.

(ii) We initially show that $x\,(s;0,0)$ solves the following maximization program:

$$\max_{x_1,..,x_N} \mathbb{E}\left[TS|\, s\right], \tag{46}$$

where s denotes a realization of the vector of private signals and

$$\mathbb{E}\left[TS|\, s\right] = \sum_{i=1}^N \left(\alpha-\mathbb{E}\left[\theta_i|s\right]\right)x_i - \frac{1}{2}\left(\beta\left(\sum_{i=1}^N x_i\right)^2+\lambda\sum_{i=1}^N x_i^2\right).$$

Hence, the FOC from the previous optimization problem are given by

$$\left(\alpha-\mathbb{E}\left[\theta_i|s\right]\right) - \left(\beta\left(\sum_{i=1}^N x_i\right)+\lambda x_i\right) = 0, i = 1,...,N.$$

Adding the previous equalities and isolating $\sum_{i=1}^N x_i$ in the resulting expression, we get

$$\sum_{i=1}^N x_i = \frac{N\alpha-\sum_{i=1}^N \mathbb{E}\left[\theta_i|s\right]}{N\beta+\lambda}.$$

Substituting the previous expression in the FOC and, then, isolating x_i, it follows that

$$x_i = \frac{\alpha}{\lambda + N\beta} + \frac{\frac{N\beta}{\lambda+N\beta}\frac{\sum_{i=1}^{N}\mathbb{E}[\theta_i|s]}{N} - \mathbb{E}\left[\theta_i|s\right]}{\lambda}. \tag{47}$$

Tedious computations yield that

$$\mathbb{E}\left[\theta_i|s\right] = \overline{\theta} + \frac{(1-\rho)\,\sigma_\theta^2}{\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)}\left(s_i - \overline{\theta}\right) + \frac{\rho\sigma_\theta^2\sigma_\varepsilon^2}{\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho\,(N-1))\,\sigma_\theta^2+\sigma_\varepsilon^2\right)}\left(\sum\nolimits_{j=1}^{N} s_j - N\overline{\theta}\right).$$

Plugging this expression into (47) and after some algebra, we have $x_i = \widehat{b}^* - \widehat{a}^* s_i + \widehat{c}^*\widetilde{s}$. Combining this result and the first part of this propositon, we have that the critical point of the optimization problem given in (46) is $x_i\,(s;0,0)$. Given that the objective function of the previous optimization problem is a concave function, we can conclude that this critical point is a global maximum.

Next, we consider a Taylor series expansion of $\mathbb{E}\left[TS|\,s\right]$ around $x\,(s;0,0)$, stopping at the second term due to the fact that $\mathbb{E}\left[TS|\,s\right]$ is quadratic, and we obtain

$$\begin{aligned}&\mathbb{E}\left[TS|\,s\right]\left(x\left(s;d,\chi\right)\right) = \mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right) + \\ &\nabla\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right)\left(\left(x\left(s;d,\chi\right)\right) - x\left(s;0,0\right)\right) + \\ &\tfrac{1}{2}\left(\left(x\left(s;d,\chi\right)\right) - x\left(s;0,0\right)\right)' D^2\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right)\left(\left(x\left(s;d,\chi\right)\right) - x\left(s;0,0\right)\right),\end{aligned}$$

where $\nabla\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right)$ and $D^2\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right)$ are, respectively, the gradient and the Hessian matrix of $\mathbb{E}\left[TS|\,s\right]$ evaluated at $x\,(s;0,0)$. Because of the optimality of $x\,(s;0,0)$ previously shown, it follows that

$$\nabla\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right) = \left(0,...,0\right).$$

In addition,

$$D^2\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right) = -\begin{pmatrix} \beta+\lambda & \beta & \cdots & \cdots & \beta \\ \beta & \beta+\lambda & \beta & \cdots & \beta \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \beta & \cdots & \beta & \beta+\lambda & \beta \\ \beta & \cdots & \cdots & \beta & \beta+\lambda \end{pmatrix}.$$

Hence,

$$\begin{aligned}&\mathbb{E}\left[TS|\,s\right]\left(x\left(s;0,0\right)\right) - \mathbb{E}\left[TS|\,s\right]\left(x\left(s;d,\chi\right)\right) = \\ &\tfrac{1}{2}\left(\beta\left(\sum\nolimits_{j=1}^{N}\left(x_j\left(s;d,\chi\right) - x_j\left(s;0,0\right)\right)\right)^2 + \lambda\sum\nolimits_{j=1}^{N}\left(x_j\left(s;d,\chi\right) - x_j\left(s;0,0\right)\right)^2\right).\end{aligned}$$

Let $\mathbb{E}\left[DWL|s\right]$ represent the expected deadweight loss in equilibrium conditional to s, which is equal to the difference between the conditional expected total surplus at the efficient allocation

$x(s;0,0)$ and the conditional expected total surplus at the equilibrium allocation $x(s;d,\chi)$. From the previous formula we have that

$$\mathbb{E}[DWL|s] = \frac{1}{2}\left(\beta N^2 \left(\widetilde{x}(s;d,\chi) - \widetilde{x}(s;0,0)\right)^2 + \lambda \sum\nolimits_{j=1}^{N} \left(x_j(s;d,\chi) - x_j(s;0,0)\right)^2\right),$$

where $\widetilde{x}(s;d,\chi) = \frac{\sum_{i=1}^{N} x_i(s;d,\chi)}{N}$ and $\widetilde{x}(s;0,0) = \frac{\sum_{i=1}^{N} x_i(s;0,0)}{N}$. In addition, direct computations yield that the previous expression is equivalent to

$$\begin{aligned}\mathbb{E}[DWL|s] &= \frac{N(N\beta+\lambda)}{2}\left(\widetilde{x}(s;d,\chi) - \widetilde{x}(s;0,0)\right)^2 \\ &+ \frac{\lambda}{2}\sum\nolimits_{j=1}^{N}\left(u_j(s;d,\chi) - u_j(s;0,0)\right)^2.\end{aligned}$$

Finally, taking expectations, we obtain (16).

Proof of Proposition 7: According to (16), it follows that $\mathbb{E}[DWL] = AI + DI$, with

$$\begin{aligned}AI &= \frac{N(N\beta+\lambda)}{2}\mathbb{E}\left[\left(\widetilde{x}(s;d,\chi) - \widetilde{x}(s;0,0)\right)^2\right] \text{ and} \\ DI &= \frac{\lambda}{2}\sum_{j=1}^{N}\mathbb{E}\left[\left(u_j(s;d,\chi) - u_j(s;0,0)\right)^2\right],\end{aligned}$$

where d is the value of price impact in equilibrium. Since this value depends on χ, in what follows let us explicitly write this dependence as $d(\chi)$. Thus,

$$\begin{aligned}AI &= \frac{N(N\beta+\lambda)}{2}\mathbb{E}\left[\left(\widetilde{x}(s;d(\chi),\chi) - \widetilde{x}(s;0,0)\right)^2\right] \text{ and} \\ DI &= \frac{\lambda}{2}\sum_{j=1}^{N}\mathbb{E}\left[\left(u_j(s;d(\chi),\chi) - u_j(s;0,0)\right)^2\right].\end{aligned}$$

(i) In relation to AI, note that

$$sign\left(\frac{\partial AI}{\partial \chi}\right) = sign\left(\frac{\partial}{\partial \chi}\mathbb{E}\left[\left(\widetilde{x}(s;d(\chi),\chi) - \widetilde{x}(s;0,0)\right)^2\right]\right).$$

In addition,

$$\mathbb{E}\left[\left(\widetilde{x}(s;(\chi),\chi) - \widetilde{x}(s;0,0)\right)^2\right] = \left(\mathbb{E}\left[\left(\widetilde{x}(s;(\chi),\chi) - \widetilde{x}(s;0,0)\right)\right]\right)^2 + var\left[\widetilde{x}(s;(\chi),\chi) - \widetilde{x}(s;0,0)\right].$$

Using (10) and (13), we have

$$x_i(s;d(\chi),\chi) = \frac{\alpha - \overline{\theta}}{d(\chi)+\lambda+N\beta} + a(d(\chi),\chi)\left(\widetilde{s} - s_i\right) - \frac{A(d(\chi),\chi)}{N\beta}\left(\widetilde{s} - \overline{\theta}\right),$$

where the expressions of $a(d(\chi),\chi)$ and $A(d(\chi),\chi)$ are given in Equation (4) and (9), respectively. Hence,

$$\widetilde{x}(s;d(\chi),\chi) = \frac{\alpha - \overline{\theta}}{d(\chi)+\lambda+N\beta} - \frac{A(d(\chi),\chi)}{N\beta}\left(\widetilde{s} - \overline{\theta}\right),$$

which implies

$$
\begin{aligned}
\left(\mathbb{E}\left[\left(\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right)\right]\right)^2 &= \frac{(d(\chi))^2\left(\alpha-\overline{\theta}\right)^2}{(\lambda+N\beta)^2\left(d(\chi)+\lambda+N\beta\right)^2} \text{ and}\\
var\left[\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right] &= \frac{\left(A(d(\chi),\chi)-A(0,0)\right)^2}{N^2\beta^2} var\left[\widetilde{s}\right].
\end{aligned}
$$

Therefore, $\left(\mathbb{E}\left[\left(\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right)\right]\right)^2$ depends on χ through the price impact (d) and it is increasing in d. Combining this fact with Corollary 1, we get that $\left(\mathbb{E}\left[\left(\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right)\right]\right)^2$ decreases with cursedness. Moreover, given that $A(d(\chi),\chi)-A(0,0)<0$, the previous expression of $var\left[\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right]$ implies that $var\left[\widetilde{x}\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;0,0\right)\right]$ increases (decreases) with cursedness if and only if the coefficient A decreases (increases) with cursedness. Using the proof of Proposition 3, the results stated in the statement of this proposition follows.

(ii) In relation to DI, note that

$$
sign\left(\frac{\partial DI}{\partial\chi}\right) = sign\left(\frac{\partial}{\partial\chi}\mathbb{E}\left[\left(u_i\left(s;d(\chi),\chi\right)-u_i\left(s;0,0\right)\right)^2\right]\right),
$$

for a given i, $i=1,...,N$. After some algebra, we have

$$
\begin{aligned}
u_i\left(s;d(\chi),\chi\right) &= x_i\left(s;d(\chi),\chi\right)-\widetilde{x}\left(s;d,\chi\right) = a(d(\chi),\chi)\left(\widetilde{s}-s_i\right) \text{ and}\\
u_i\left(s;0,0\right) &= x_i\left(s;0,0\right)-\widetilde{x}\left(s;0,0\right) = a(0,0)\left(\widetilde{s}-s_i\right).
\end{aligned}
$$

Hence, $\mathbb{E}\left[\left(u_i\left(s;d(\chi),\chi\right)-u_i\left(s;0,0\right)\right)^2\right] = \left(a(d(\chi),\chi)-a(0,0)\right)^2 var\left[\widetilde{s}-s_i\right]$. Thus,

$$
sign\left(\frac{\partial}{\partial\chi}\mathbb{E}\left[\left(u_i\left(s;d(\chi),\chi\right)-u_i\left(s;0,0\right)\right)^2\right]\right) = sign\left(\frac{\partial}{\partial\chi}\left(a(d(\chi),\chi)-a(0,0)\right)^2\right)
$$

Therefore,

$$
\frac{\partial}{\partial\chi}\left(a(d(\chi),\chi)-a(0,0)\right)^2 = 2\left(a(d(\chi),\chi)-a(0,0)\right)\frac{\partial}{\partial\chi}a(d(\chi),\chi)
$$

From Corollary 1, we know that $\frac{\partial}{\partial\chi}a(d(\chi),\chi)>0$. Hence,

$$
sign\left(\frac{\partial}{\partial\chi}\left(a(d(\chi),\chi)-a(0,0)\right)^2\right) = sign\left(a(d(\chi),\chi)-a(0,0)\right).
$$

Direct computations yield

$$
a(d(\chi),\chi)-a(0,0) = \frac{\chi\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}+(1-\chi)\frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda} - \frac{\frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}}{\lambda} = \frac{(1-\rho)\,\sigma_\theta^2 G(\chi)}{\left(d(\chi)+\lambda\right)\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)}
$$

with $G(\chi)=\frac{\chi\rho\sigma_\varepsilon^2}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)}-\frac{d(\chi)}{\lambda}$. Given that d decreases with χ, we have that $G(\chi)$ is an increasing function in χ. Moreover,

$$
G(0) = -\frac{d(0)}{\lambda} < 0 \text{ and } \lim_{\chi\to 1} G(\chi) = \frac{\rho\sigma_\varepsilon^2}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)} - \frac{\beta\left(-N+2\right)-\lambda+\sqrt{\left(\beta\left(N-2\right)+\lambda\right)^2+4\beta\lambda}}{2\lambda}.
$$

Notice that $\lim_{\chi\to 1} G(\chi)$ as a function of ρ is an increasing function. Moreover, $\underset{\rho\to 0}{lim}\left(\lim_{\chi\to 1} G(\chi)\right) < 0$ and $\underset{\rho\to 1}{lim}\left(\lim_{\chi\to 1} G(\chi)\right) = \infty$. Thus, there exists a value $\widehat{\rho}_{DI}$, which is the solution of $\frac{\rho\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)(1-\rho)} = \frac{\beta(-N+2)-\lambda+\sqrt{(\beta(N-2)+\lambda)^2+4\beta\lambda}}{2\lambda}$, such that if $\rho \le \widehat{\rho}_{DI}$, then $\lim_{\chi\to 1} G(\chi) \le 0$. Given that $G(\chi)$ is an increasing function in χ, we conclude that $G(\chi) < 0$ for all χ, and hence, $\frac{\partial DI}{\partial\chi} < 0$ for all χ. Otherwise, if $\rho > \widehat{\rho}_{DI}$, then $\lim_{\chi\to 1} G(\chi) > 0$. In this case, $G(\chi) < 0$, and hence, $\frac{\partial DI}{\partial\chi} < 0$, provided that χ is low enough. By contrast, when χ is high enough, $G(\chi) > 0$, and hence, $\frac{\partial DI}{\partial\chi} > 0$.

Proof of Corollary 7:
(i) Combining (4), (9), (16), (17), (18), (33), and (34), and taking into account that under perfect competition we have that $d = 0$, we obtain

$$\mathbb{E}[DWL] = \frac{\chi^2\rho^2\sigma_\theta^4\sigma_\varepsilon^4(N-1)^2}{2\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2(\lambda+N\beta)} + \frac{\chi^2(N-1)\sigma_\theta^4\lambda^2\rho^2\sigma_\varepsilon^4}{2\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2\lambda^3},$$

which implies that $\mathbb{E}[DWL]$ increases with χ.

(ii) When β converges to infinity and $\frac{\alpha}{\beta}$ converges to Q, $\underset{\beta\to\infty}{lim}\,\widetilde{x}(s;0,0) = \underset{\beta\to\infty}{lim}\,\widetilde{x}(s;d,\chi) = \frac{Q}{N}$, which implies that in this limiting case the aggregate inefficiency vanishes. With respect to expected distributive inefficiency, the equation that characterizes $\widehat{\rho}_{DI}$ becomes $\frac{\rho\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)(1-\rho)} = \frac{1}{N-2}$, which implies that $\widehat{\rho}_{DI} = \frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$.

Proof of Proposition 8: When the demand is perfectly inelastic, the optimal quantities and price, in equilibrium, satisfy

$$x_i = \frac{Q}{N} + \frac{(1-\chi)\frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2} + \chi\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}{d+\lambda}\left(\widetilde{s}-s_i\right) \text{ and} \tag{48}$$

$$p = \overline{\theta} + \frac{(d+\lambda)Q}{N} + \sigma_\theta^2\frac{(1+\rho(N-1))\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\chi\rho\sigma_\varepsilon^2(N-1)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1+\rho(N-1))\sigma_\theta^2+\sigma_\varepsilon^2\right)}\left(\widetilde{s}-\overline{\theta}\right). \tag{49}$$

Direct computations yield

$$\mathbb{E}[\pi_i] = \mathbb{E}[p-\theta_i-\frac{\lambda}{2}x_i]\mathbb{E}[x_i] + cov[p-\theta_i-\frac{\lambda}{2}x_i, x_i].$$

Concerning the first term of $\mathbb{E}[\pi_i]$, we have

$$\mathbb{E}[p-\theta_i-\frac{\lambda}{2}x_i]\mathbb{E}[x_i] = \left(\frac{(d+\lambda)Q}{N} - \frac{\lambda}{2}\frac{Q}{N}\right)\frac{Q}{N} = \frac{2d+\lambda}{2}\frac{Q^2}{N^2}.$$

In relation to the second term of $\mathbb{E}[\pi_i]$, note that

$$cov[p-\theta_i-\frac{\lambda}{2}x_i, x_i] = cov[p-\theta_i, x_i] - \frac{\lambda}{2}var[x_i].$$

In addition, using the expression of (48) and (49), we have

$$cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]=\frac{(N-1)\,\sigma_\theta^4\left(\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)+\chi\rho\sigma_\varepsilon^2\right)\left((2d+\lambda)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\chi\lambda\rho\sigma_\varepsilon^2\right)}{2N\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2(d+\lambda)^2\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)}.$$

Differentiating,

$$\frac{\partial}{\partial\chi}cov[p-\theta_i-\tfrac{\lambda}{2}x_i,x_i]=\frac{\sigma_\theta^4(N-1)\left(d\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\lambda\chi\rho\sigma_\varepsilon^2\right)}{N(d+\lambda)^2\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}\left(\sigma_\varepsilon^2\rho+\frac{\left((1-\rho)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)+\chi\rho\sigma_\varepsilon^2\right)}{d+\lambda}\left(-\frac{\partial d}{\partial\chi}\right)\right).$$

Therefore, $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$ is equivalent to

$$d\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\lambda\chi\rho\sigma_\varepsilon^2>0.$$

Using the expression of d given in Corollary 3, we get

$$d\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\lambda\chi\rho\sigma_\varepsilon^2=\frac{\lambda K(\chi)}{\left((1+\rho\,(N-1))\,\sigma_\theta^2+\sigma_\varepsilon^2\right)(N-2-M)},$$

with

$$K(\chi)=\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho+N\rho)-\rho\sigma_\varepsilon^2\,(N-1)\left(2\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)+N\rho\sigma_\theta^2\right)\chi.$$

As $N-2-M>0$, we know that

$$sign\left(d\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\lambda\chi\rho\sigma_\varepsilon^2\right)=sign\left(K(\chi)\right).$$

Moreover, note that
$N-2-M=\frac{\rho\sigma_\varepsilon^2(N-1)\left(\sigma_\theta^2(\rho(N-2)+2)+2\sigma_\varepsilon^2\right)\chi-\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((2\rho(N-1)-N+2)\sigma_\varepsilon^2-(1-\rho)(N-2)(1+\rho(N-1))\sigma_\theta^2\right)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2+N\rho\sigma_\theta^2\right)\left(\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)+\chi\rho\sigma_\varepsilon^2\right)}.$

Then, we can distinguish two cases:

1) $(2\rho\,(N-1)-N+2)\,\sigma_\varepsilon^2-(1-\rho)\,(N-2)\,(1+\rho\,(N-1))\,\sigma_\theta^2<0$ and

2) $(2\rho\,(N-1)-N+2)\,\sigma_\varepsilon^2-(1-\rho)\,(N-2)\,(1+\rho\,(N-1))\,\sigma_\theta^2\geq 0$.

Case 1: If $(2\rho\,(N-1)-N+2)\,\sigma_\varepsilon^2-(1-\rho)\,(N-2)\,(1+\rho\,(N-1))\,\sigma_\theta^2<0$ (which is satisfied $\rho<\widehat{\rho}<1$), then $N-2-M>0$ for all χ.

Given that $K(\chi)$ decreases with χ and

$$K(1)=\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\left(\sigma_\theta^2+(N-1)\,\sigma_\varepsilon^2\right)\right)\left((1-\rho+N\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right),$$

it follows that

if $\rho<\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$, then $K(\chi)>0$ for all χ, which implies $d\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)(1-\rho)-\lambda\chi\rho\sigma_\varepsilon^2>0$ and, therefore, $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$;

if $\rho\geq\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$, then $K(\chi)>0$ if and only if $\chi<\overline{\chi}$, with $\overline{\chi}=\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)(1+\rho(N-1))}{\rho\sigma_\varepsilon^2(N-1)\left(2\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)+N\rho\sigma_\theta^2\right)}$, which implies that

$$\frac{\partial}{\partial\chi}cov[p-\theta_i-\tfrac{\lambda}{2}x_i,x_i]>0 \text{ if and only if } \chi<\overline{\chi}.$$

Case 2: If $(2\rho(N-1)-N+2)\,\sigma_\varepsilon^2-(1-\rho)(N-2)(1+\rho(N-1))\,\sigma_\theta^2\geq 0$ (which is satisfied $1\geq\rho\geq\widehat{\rho}$), then $N-2-M>0$ if and only if $\chi>\underline{\chi}$, with

$$\underline{\chi}=\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left((2\rho(N-1)-N+2)\sigma_\varepsilon^2-(1-\rho)(N-2)(1+\rho(N-1))\sigma_\theta^2\right)}{\rho\sigma_\varepsilon^2(N-1)\left((2+\rho(N-2))\sigma_\theta^2+2\sigma_\varepsilon^2\right)}.$$

Moreover,

if $\rho<\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$, then $K(\chi)>0$ $\left(\text{and therefore, } \frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0\right)$ for all the feasible values of χ.

if $\rho\geq\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$, then $K(\chi)>0$ $\left(\text{and therefore, } \frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0\right)$ if and only if

$$\underline{\chi}<\chi<\overline{\chi}. \tag{50}$$

To sum up, we have that

if $\rho<\min\left\{\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2},\widehat{\rho}\right\}$,[14] then $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$ for all the feasible values of χ (i.e., $\chi\in[0,1]$);

if $\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}<\rho<\widehat{\rho}$, then $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$ if and only if $\chi<\overline{\chi}$;

if $\widehat{\rho}<\rho<\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}$, then $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$ for all the feasible values of χ $\left(\text{i.e., } \chi>\underline{\chi}\right)$; and

if $\rho>\max\left\{\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2},\widehat{\rho}\right\}$, then $\frac{\partial}{\partial\chi}cov[p-\theta_i-\frac{\lambda}{2}x_i,x_i]>0$ if and only if (50) holds.

Therefore, we conclude that if Q is high enough, then $\mathbb{E}[\pi_i]$ decreases with χ. Otherwise, i.e., if Q is low enough, the opposite result holds provided that ρ is low enough or χ is low enough and $\rho<1$.

Remark: It can be seen that when $\rho=1$ there is no value of χ that satisfies (50). In this case the second term of expected profits decreases with profits for all values of χ.

[14] It can be seen that when $N\leq 3$, $\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}<\widehat{\rho}$. Otherwise, when $N>3$, $\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+(N-1)\sigma_\varepsilon^2}>\widehat{\rho}$.